Errata for Quantum Mechanics
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Preface

• Page xv: Add a sentence at the end of the next-to-last paragraph:

Rudaz (University of Minnesota), and several anonymous reviewers. My special
thanks to Kenneth Lane and his students at Boston University.

I thank the Department of Physics and Astronomy at UCLA for granting me

Chapter I

• Page 14, Equation (1.78): The last expression needs an $\epsilon$ in front:

$$G = \sum_{m,i} p_{m,i} \delta r_{m,i} = \epsilon \sum_{ijk} \epsilon_{ijk} p_{m,i} \hat{n}_{m,k} = \epsilon \sum_m (r_m \times p_m) \cdot \hat{n} = \epsilon L \cdot \hat{n} \quad (1.78)$$

Chapter II

• Page 22, Equation (2.2): In the leftmost expression, change $\hbar$ to $h$:

$$\frac{\hbar}{p} = \lambda = \frac{2\pi \hbar}{\sqrt{2mE}} \approx 1.2 \, \text{Å} \quad (2.2)$$

• Page 25, Equation (2.6a): Change the last $\alpha$ to $\beta$:

$$\left[ \alpha + \beta \right] |\psi\rangle = \alpha |\psi\rangle + \beta |\psi\rangle \quad (2.6a)$$

Chapter III

• Page 75, Equation (3.80): Put an $\hbar$ in front of the left-hand side:

$$\hbar [r_i, L_j] = i\hbar \sum_k \epsilon_{ijk} r_k \quad (3.80)$$

• Page 84, Equation (3.137): The last term should be $Ze^2/r$:

$$H = \frac{p^2}{2m} - \frac{Ze^2}{r} \quad (3.137)$$

• Page 90, Equation (3.172). Change $m$ to $M$.

$$n - l - 1 = M \quad (3.172)$$
Chapter IV

- Page 122, just above Equation (4.105), insert “solutions” between “the” and “to”:

The eigenstates are linear combinations of $|1, -1/2\rangle$ and $|0, 1/2\rangle$. The eigenvalues are the solutions to the characteristic equation:

- Page 126, third paragraph, third line: change $1/2f$ to $1/2$:

  Unless $l = 0$, there are two states with $m = l - 1/2$. One linear combination will be in the $j = l + 1/2$ ladder. The remaining one must be the top of a new ladder, this time with $j = l - 1/2$. The number of states in these two ladders adds

- Page 131, Problem 4.2, part (b): Change the superscript $j$ to 1:

  \[ \sum_{mn} U_{am} D^{(1)}(\tilde{J}_i)_{mn} U_{nb}^\dagger = (\tilde{J}_i)_{ab} \]

- Page 132, Problem 4.4, part (c). The last equation on the page should be changed to

  \[ \psi(r) = R^m_{\tilde{J}i}(r)Y^{m\pm}_{l} (\theta, \phi) \]

Chapter V

- Page 140, line below Equation (5.6): Change $g(a)$ to $g(\epsilon)$:

  For a one-parameter subgroup of continuous symmetries $g(\epsilon) = \exp(-i\epsilon G)$,

- Page 161, Equation (5.123): Divide the exponent by $\hbar$:

  \[ U(t_f, t_i) = e^{-iH(t_f-t_i)/\hbar} \]

  \[ (5.123) \]

Chapter VII

- Page 203, Equation (7.8): In the first line change $E_o^n$ to $E_o^n$ (twice):

  \[ (H - E_o^n + E_o^n - H_o) |\psi_n\rangle = H' |\psi_n\rangle \]

  \[ (H_o - E_o^n) |\psi_n\rangle = (\Delta_n - H') |\psi_n\rangle \]

  \[ (7.8) \]

- Page 205, second bulleted paragraph, seventh line: Change “so” to “but”:

  Two states in one of these ladders will have the same unperturbed energy, but we

- Page 214, Equations (7.69) to (7.70): Change the subscript “3” to “D”:

  \[ H_D = -\frac{1}{8m^2} \nabla^2 V(r) \]

  \[ E_D = \frac{\alpha^4 m}{2n^3 \Delta_{l0}} \]

  \[ (7.69) \]

  \[ (7.70) \]
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• Page 226. Equation (7.133): Delete the subscript “i” (four times):

\[
\int \psi_{100}(r) \left( \frac{p^2}{2m} - \frac{\sigma e^2}{r} \right) \psi_{100}(r) d^3 r = E_1^\sigma(\sigma) = \sigma^2 E_1^\sigma \tag{7.133}
\]

Chapter VIII

• Page 270, Equation (8.40): change the left-hand side to \( f^{(1)}(\Theta) \):

\[
f^{(1)}(\Theta) = -\frac{1}{4\pi} \int e^{i\mathbf{q} \cdot \mathbf{r}} U(r) d^3 r
\]
\[
= \frac{1}{2} \int_0^\infty r^2 dr \int_{-1}^1 e^{iqr \cos \theta} U(r) d\cos \theta = -\frac{1}{q} \int_0^\infty r \sin(qr) U(r) dr
\]

• Page 270, Equation (8.39), replace \( r \) by \( r' \) inside the integral.

\[
f^{(1)}(\theta, \phi) = -\frac{1}{4\pi} \int e^{i(k-k') \cdot \mathbf{r}'} U(r') d^3 r'
\]

• Page 273, Equation (8.56): In the second line, second term inside the large parentheses, move the asterisk from \( \chi \) to \( \phi \). In the fourth line, first expression, second term inside the parentheses, change plus to minus:

\[
\frac{k \sigma_{tot}}{(2\pi)^3} = -\operatorname{Im} \int \left( \phi^* \frac{\partial \chi}{\partial r} + \chi^* \frac{\partial \phi}{\partial r} \right) r^2 d\Omega
\]
\[
= -\operatorname{Im} \int \left( \phi^* \frac{\partial \chi}{\partial r} - \chi^* \frac{\partial \phi}{\partial r} \right) r^2 d\Omega
\]
\[
= -\operatorname{Im} \int \left( \phi^* \frac{\partial \psi}{\partial r} - \psi^* \frac{\partial \phi}{\partial r} \right) r^2 d\Omega
\]
\[
= -\operatorname{Im} \int \left( \phi^* \nabla \psi - \psi^* \nabla \phi \right) \cdot \mathbf{n} dS = -\operatorname{Im} \int \left( \phi^* \nabla^2 \psi - \psi^* \nabla^2 \phi \right) d^3 r
\]
\[
= -\operatorname{Im} \int \phi^* (r) U(r) \psi(r) d^3 r = \frac{1}{2\pi^2} \operatorname{Im} f(0) \tag{8.56}
\]

• Page 277, Equation (8.80), in the first line, in the numerator of first factor, change \( 2l + l \) to \( 2l + 1 \):

\[
A_l \rho^l \frac{2l!}{(2l + 1)!} = \frac{2l + 1}{2} \sum_{n=0}^l \frac{i^n \rho^n}{n!} \frac{2n(n)!^2}{(2n)!} \int_0^1 P_n(z) P_l(z) dz
\]
\[
= \frac{1}{2} \frac{2l!(l)!^2}{l!(2l)!} \tag{8.80}
\]

• Page 280, Equation (8.97): Change the last \( r \) to \( r' \). Move the asterisk to after the second \( Y^m_l \) factor.
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- Page 280, Equation (8.98) Change the arguments of the first $Y_l^m$ to $(\theta', \phi')$.
- Page 280, Equation (8.99) second line, change $\hat{n}$ to $\theta', \phi'$, and change $\hat{n}_z$ to 0, $\phi$:

$$\phi_k(r') = \frac{4\pi}{(2\pi)^2} \sum_{l,m} i^l Y_l^m(\theta, \phi) Y_l^m(\theta', \phi')^* j_l(\kappa r')$$  \hspace{1cm} (8.97)

and

$$\psi_k(r') = \frac{4\pi}{(2\pi)^2} \sum_{l,m} i^l Y_l^m(\theta', \phi')^* Y_l^m(0, \phi) R_l(r')$$ \hspace{1cm} (8.98)

Therefore the scattering amplitude is

$$f(\theta, \phi) = -4\pi \int \left( \sum_{l,m} (-i)^l Y_l^m(\theta, \phi) Y_l^m(\theta', \phi') j_l(\kappa r') \right)$$

$$\times U(r') \left( \sum_{l'} i^{l'} Y_l^{m'}(\theta', \phi') R_{l'}(r') Y_{l'}^m(0, \phi) \right) r'^2 dr' d\Omega'$$ \hspace{1cm} (8.99)

$$= -\int_0^\infty \left[ \sum_{l} (2l + 1) j_l(\kappa r) U(r) R_l(r) P_l(\cos \theta) \right] r'^2 dr'$$

- Page 280, Equation (8.100) On the left, $k$ should be the denominator of all the rest, not just $\delta_l$:

$$\frac{e^{i\delta_l} \sin \delta_l}{k} = -\int_0^\infty j_l(\kappa r) U(r) R_l(r) r^2 dr$$ \hspace{1cm} (8.100)

- Page 280, last sentence, replace “in that number” by “in that limit”.

- Page 284, Problem 8.1, in the line just following the unnumbered displayed equation, on the right-hand-side of the in-line equation, remove the middle vertical line:

where $\rho(r) = |\psi_{100}(r)|^2$, the probability distribution of the bound electron.

- Page 287, Reference [3], Replace the comma after K by a period:

[3] K. Gottfried, Quantum Mechanics, Volume I, Benjamin, 1966. See also Appendix D.

Chapter IX

- Page 288, in the subsubsection heading just below the 9.1.2 subsection heading, change Transitions to Transitions.

- Page 289, Equation (9.10), second line, $4m^2$ in denominator should be just $m^2$.

$$P_b(t) \approx |V_{ba}|^2 \left| \frac{e^{i(\omega_{ba} - \omega)t} - 1}{\omega_{ba} - \omega} \right|^2$$

$$= \frac{\alpha}{m^2} A^2 |\langle \psi_b | \hat{p} \cdot \hat{e} e^{-i \hat{k} \cdot \hat{r}} | \psi_a \rangle|^2 \left| \frac{\sin(\omega_{ba} - \omega)t/2}{|\omega_{ba} - \omega|} \right|^2$$ \hspace{1cm} (9.10)
• Page 290, Equation (9.16), replace \(4m^2\) by \(m^2\) and \(8\pi^2\) by \(4\pi^2\).

\[
P(t) \rightarrow \alpha \frac{4\pi^2}{m^2\omega_{ba}} \rho(\omega_{ba}) \left| \langle \psi_{ba} | \mathbf{p} \cdot \hat{\mathbf{e}} e^{-ikr} | \psi_{ba} \rangle \right|^2 t \tag{9.16}
\]

• Page 291, Equation (9.17), replace \(4m^2\) by \(m^2\) and \(8\pi^2\) by \(4\pi^2\).

\[
\Gamma_b = \frac{\alpha}{m^2\omega_{ba}} \rho(\omega_{ba}) \left| \langle \psi_{ba} | \mathbf{p} \cdot \hat{\mathbf{e}} e^{-ikr} | \psi_{ba} \rangle \right|^2 \tag{9.17}
\]

• Page 291, in the line just above Equation (9.21) change \(\theta\) to \(\cos^2 \theta\). In Equation (9.21) change \(\cos \theta\) to \(\cos^2 \theta\): where \(\theta\) is the angle between the constant vector \(\mathbf{r}\) and the polarization. If the radiation is unpolarized and isotropic, just replace \(\cos^2 \theta\) by its average value:

\[
\cos^2 \theta \rightarrow \frac{1}{4\pi} \int \cos^2 \theta \, d\Omega = \frac{1}{3} \tag{9.21}
\]

• Page 292, change the text between Equations (9.25) and (9.26) to read: “Then \(a_{ba}(\omega)\) is related to \(A_{ba}(\omega)\) by”

• Page 294: There is no Equation (3.33). Latex just skipped an equation number! I don’t really understand how this could have happened, but it is under control.

• Page 294, in the text just below Equation (9.30), change “has an inverse” to “exists”:

Even though the operator \(H - \omega\) has no inverse for positive real \(\omega\), \(G(\omega)\) exists for real \(\omega\) and any finite real \(\epsilon\).

• Page 295, in the sentence above equation (9.42), change the first (9.40) to (9.39):

From equations (9.39) and (9.40) it also follows that

• Page 301, just before the “Optical Theorem” heading, add “The relativistically correct forms of Equations (9.71) and (9.72) are obtained replacing \(m^2\) by \(\omega_a^2\).”

• Page 304, Equation (9.90): Change the left hand side to \(\langle \phi_b | T | \phi_a \rangle\):

\[
\langle \phi_b | T | \phi_a \rangle = \delta_3 (\mathbf{K} - \mathbf{K}) \langle \phi_b | \bar{T} | \phi_a \rangle \tag{9.90}
\]

• Page 304, Equation (9.93): The integrals should end in \(d^3k_1'd^3k_2'\) and \(d^3K'd^3k'\):

\[
\bar{\Gamma} = 2\pi \int |T_{ba}|^2 \delta (\omega_a - \omega_b) d^3k_1'd^3k_2' = 2\pi \int |\delta_3 (\mathbf{K}' - \mathbf{K})\bar{T}_{ba}|^2 \delta (\omega_a - \omega_b) d^3K'd^3k' \approx 2\pi \int |\bar{T}_{ba}|^2 \delta (\omega_a - \omega_b)\delta_3 (\mathbf{K}' - \mathbf{K}) \frac{V}{(2\pi)^3} d^3K'd^3k' \tag{9.93}
\]
• Pages 304-305, In equations (9.90) through (9.95) change $T'$ to $T$ everywhere.

• Page 306, Equation (9.103). In the first line exchange $r_1$ and $r_2$ in the exponent. In the second line change the sign of the exponent:

$$H'_{ba,2} = \frac{1}{2} \frac{1}{(2\pi)^3} \int V(|r_1 - r_2|) e^{i(k_2-k'_1) \cdot r_2} e^{i(k_1-k'_2) \cdot r_1} d^3 r_1 d^3 r_2$$

$$= \frac{1}{2} \frac{1}{(2\pi)^3} \int V(r) e^{i(k+k') \cdot r} \delta_3 (K - K') d^3 r$$

(9.103)

• Page 307: In equation (9.104) delete the delta function $\delta_3 (K - K')$ in the first line. Change $T$ to $\tilde{T}$ everywhere in (9.104) and (9.105). And replace $p$ by $k$ in these and the next two equations:

$$\tilde{T}_{\text{direct}} (\omega_a, \theta, \phi) = \frac{1}{(2\pi)^3} \int V(r)e^{i(k-k') \cdot r} d^3 r$$

$$= \frac{e^2}{2\pi^2 q^2} = \frac{e^2}{4\pi^2 k^2 (1 - \cos \theta)} = \frac{\alpha}{8\pi^2 k^2 \sin^2 (\theta/2)}$$

(9.104)

$$\tilde{T}_{\text{exchange}} (\omega_a, \theta, \phi) = \tilde{T}_{\text{direct}} (\omega_a, \pi - \theta, \phi + 2\pi)$$

$$= \frac{e^2}{16\pi^2 k^2 \cos^2 (\theta/2)} = \frac{\alpha}{8\pi^2 k^2 \cos^2 (\theta/2)}$$

(9.105)

$$\frac{d\sigma}{d\Omega} = (2\pi)^4 \mu^2 \frac{\alpha^2}{64\pi^4 k^2} \left| \frac{1}{\sin^2 (\theta/2)} + \frac{1}{\cos^2 (\theta/2)} \right|^2 = \frac{\alpha^2 \mu^2}{4k^2} \left| \frac{1}{\sin^2 (\theta/2)} + \frac{1}{\cos^2 (\theta/2)} \right|^2$$

$$= \frac{\alpha^2 \mu^2}{4k^2} \left( \frac{1}{\cos^2 (\theta/2)} + \frac{1}{\sin^2 (\theta/2)} \pm 2 \frac{1}{\sin^2 (\theta/2) \cos^2 (\theta/2)} \right)$$

(9.106)

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{unpolarized}} = \frac{\alpha^2 \mu^2}{4k^2} \left( \frac{1}{\sin^2 (\theta/2)} + \frac{1}{\cos^2 (\theta/2)} - \frac{1}{\sin^2 (\theta/2) \cos^2 (\theta/2)} \right)$$

(9.107)

• Page 312, Equation (9.132): On the left, change $\omega'$ to $\omega'_a$. On the right, change the denominator to $\omega_b - \omega_a$. In Equation (9.133) replace $\omega$ by $\omega_b$. In the text below Equation (9.134) delete “with $\omega'_a = \text{Re} \omega'$ above”:

$$\omega'_a = \omega_a + H'_{aa} + \text{Re} \sum_{b \neq a} |H'_{ba}|^2 \frac{1}{\omega_b - \omega_a} + \cdots$$

(9.132)

$$-\pi \sum_{b \neq a} |H'_{ba}|^2 \delta (\omega_b - \omega_a) = -\Gamma/2$$

(9.133)

$$G(\omega)_{aa} = \frac{1}{\omega - \omega'_a + i\Gamma/2}$$

(9.134)

Evidently $T(\omega)$ has a singularity in the lower half plane,
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• Page 314 just below Equation (9.142) change “closer to” to “farther from”:
  which is farther from unity than the same probability

• Page 315, Equation (9.148) change \( \text{abs}H'_{ba} \) to \( |H'_{ba}| \):
  \[
  \frac{d}{dt}|a_{ba}(t)|^2 \rightarrow |H'_{ba}|^2 \frac{2(\omega_a' - \omega_b) \sin(\omega_a' - \omega_b)t}{(\omega_a' - \omega_b)^2 + \Gamma^2/4} \quad (9.148)
  \]

Chapter X

• Page 340, Figure 10.1. The arcs should form a circle, whose center is at the base of the arrow. [Some glitch is a graphics program printed some of the arcs in the wrong place.]

• page 355, Reference [14], change Tycho to Tycko, here and in the index.

Chapter XI

• Page 365, Equations (11.57) and (11.57): In both equations change \( 4\pi \) to \( 4\pi \).

\[
E(r, t) = -\frac{\partial A}{\partial t} = \frac{(4\pi)^{1/2}}{(2\pi)^{3/2}} i \int \frac{1}{\sqrt{2\omega}} \omega [a(k, t) - a^\dagger(-k, t)] e^{ikr} d^3k \quad (11.57)
\]

and

\[
B(r, t) = \nabla \times A = -\frac{(4\pi)^{1/2}}{(2\pi)^{3/2}} i \int \frac{1}{\sqrt{2\omega}} \omega \hat{n} \times [a(-k, t) + a^\dagger(k, t)] e^{-ikr} d^3k \quad (11.58)
\]

• Page 377, Equation (11.119). Replace \( 2^5 \) by \( 2^6 \), and therefore replace \( 9.49 \times 10^{-31} \) by \( 1.87 \times 10^{-30} \) and replace \( 1.44 \times 10^{-15} \) by \( 2.84 \times 10^{-15} \). The the next line needs to read “The lifetime is \( 3.53 \times 10^{14} \) seconds, over 100,000 centuries...”. Also, delete Equation (11.117e) – there is nothing there:

The \( J = 0 \) state is

\[
\frac{|\frac{1}{2}, -\frac{1}{2}| - |\frac{1}{2}, \frac{1}{2}|}{\sqrt{2}} \quad M = 0 \quad (11.117d)
\]

\[
\Gamma = \frac{2^6}{37} a_{13}^3 g_p^3 (m/M)^3 m = 1.87 \times 10^{-30} \text{eV} = 2.84 \times 10^{-15} \text{sec}^{-1} \quad (11.109)
\]

The lifetime is \( 3.53 \times 10^{14} \) seconds, over 100,000 centuries. It should be clear

• Page 403 Problem 11.5: Delete the sentence “What is the width of these components of the Lyman-β line?”

Compute the rate for the spontaneous decay of a \( 3P \) state to the \( 1S \) state. (Here you can ignore spin—these are electric transitions.)
Chapter XII

• Page 409, Equation (12.14c): Change $x$ to $z$:

$$z' = z$$  \hspace{1cm} (12.14c)

• Page 410, Equations (12.20): The exponents should be boldface:

$$\bar{R} = e^{-i\hat{n} \cdot \hat{J}} \hspace{1cm} (12.20a)$$
$$\bar{B} = e^{-i\omega \hat{n} \cdot \hat{K}} \hspace{1cm} (12.20b)$$

• Page 416, Equation (12.67): Change the sign of the last term inside the large parentheses from minus to plus:

$$\left( \frac{1}{2m} \frac{d^2}{dr^2} - \frac{l(l+1) - \alpha^2}{2mr^2} + \frac{\omega \alpha}{mr} + \frac{\omega^2 - m^2}{2m} \right) u(r) = 0 \hspace{1cm} (12.67)$$

Chapter XIII

• Page 439, Equations (13.9) through (13.11): Change pi to $\pi$ four times:

$$\Psi(r) = \frac{1}{(2\pi)^{3/2}} \int e^{ip \cdot r} a(p) d^3p \hspace{1cm} (13.9)$$

Then

$$\Psi^\dagger(r) \rangle = \frac{1}{(2\pi)^{3/2}} \int e^{-ip \cdot r} a^\dagger(p) \rangle d^3p = \frac{1}{(2\pi)^{3/2}} \int e^{-ip \cdot r} |p\rangle d^3p \hspace{1cm} (13.10)$$

This is a one-particle state whose wave function is

$$\psi(r') = \langle r' \mid \Psi^\dagger(r) \rangle = \frac{1}{(2\pi)^{3/2}} \int e^{-ip \cdot r'} \langle r' \mid p\rangle d^3p = \delta_3(r-r') \hspace{1cm} (13.11)$$