

COMPREHENSIVE WRITTEN EXAMINATION FOR THE MASTER'S DEGREE
AND QUALIFYING EXAMINATION FOR THE PH.D. DEGREE
DEPARTMENT OF PHYSICS
Thursday, April 2, and Friday, April 3, 1998

PART I: THURSDAY, APRIL 2

Important — please read carefully.

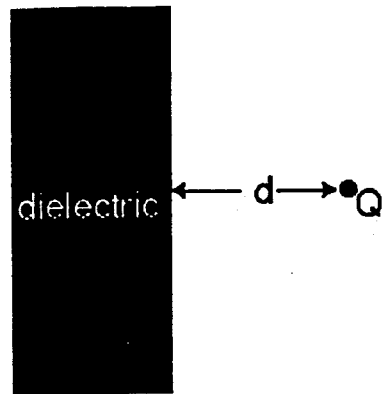
The exam (6 hours) is in two parts:

- | | |
|---------------|--|
| Part 1 | Electromagnetic Theory, Statistical Mechanics and Thermodynamics |
| April 2 | 7 Problems — DO ALL PROBLEMS. |
| 9:00-12:00 | This part will be collected at the end of three hours.
Each problem counts for 20 points; the total is 140. |
| Part 2 | Quantum Mechanics, Statistical Mechanics and Thermodynamics |
| April 3 | 7 Problems — DO ALL PROBLEMS. |
| 9:00-12:00 | This part will be collected at the end of three hours.
Each problem counts for 20 points; the total is 140. |

Instructions

- 1) This is a **closed book** exam and calculators are not to be used.
- 2) Work each problem on a separate sheet of paper. Use one side only.
- 3) Print your **name and problem number on EACH AND EVERY** page. (Note: Pages without names may not be counted.)
- 4) Return the problem page as the first page of your answer.
- 5) If a part of any question seems ambiguous to you, state clearly what your interpretation is and answer the question accordingly.

1) (E&M) A charge Q sits a distance d outside of a dielectric having dielectric constant ϵ . The surface of the dielectric may be treated as planar. (See figure.) What is the electrostatic energy of this configuration? (Do not include the energy in the field due to Q when it is isolated.)



2) (E&M) Consider a simple electromagnetic model of a time-dependent medium:

$$4\pi\vec{J} = (\epsilon(t) - 1) \frac{\partial\vec{E}}{\partial t}.$$

Assume that ϵ changes very rapidly at $t = t_0$: for $t < t_0$, $\epsilon = 1$; and for $t > t_0$, $\epsilon = \epsilon_0$. Now consider a linearly polarized electromagnetic wave propagating in this medium so that for $t > t_0$,

$$\vec{E} = E_0(x - ct) \hat{y}.$$

- Find \vec{B} for $t < t_0$.
- Show that \vec{B} , \vec{E} , and $\epsilon(t) \frac{\partial\vec{E}}{\partial t}$ are continuous at $t = t_0$.
- Write down the form of the \vec{E} field for $t > t_0$. (Hint: What polarization and phase velocity do you expect?)
- Find the full solution by connecting the $t < t_0$ solution to the $t > t_0$ solution.

3) (E&M) Consider a neutral sheet of time-dependent current which is uniform over the entire y - z plane at $x = 0$. Specifically:

$$\vec{J} = J_0 \cos(\omega t) \delta(x) \hat{y},$$

where J_0 is constant, δ is the Dirac delta function, and \hat{y} is a unit vector in the y direction.

- Write down or derive the jump in \vec{E} and \vec{B} across the $x = 0$ plane.
- Calculate \vec{E} and \vec{B} for $x > 0$ and for $x < 0$.
- Calculate the energy flux for $x > 0$, find the time averaged radiated power per unit area, and the work done by the electric field on the charge per unit area.

4) (E&M) Consider the motion of a charge in an electric field \vec{E} , using the correct equations of special relativity. Let $\vec{E} = E_0 \hat{x}$, with E_0 constant. Consider a particle with initial conditions $x = y = z = v_x = v_y = v_z = 0$ at $t = 0$, but with $v_y \neq 0$ at $t = 0$.

a) Solve for $v_x(t)$, $v_y(t)$, and $v_z(t)$. Also find simplified expressions for v_x and v_y when $t \rightarrow \infty$.

b) Find $x(t)$, $y(t)$, and find the trajectory $x(y)$.

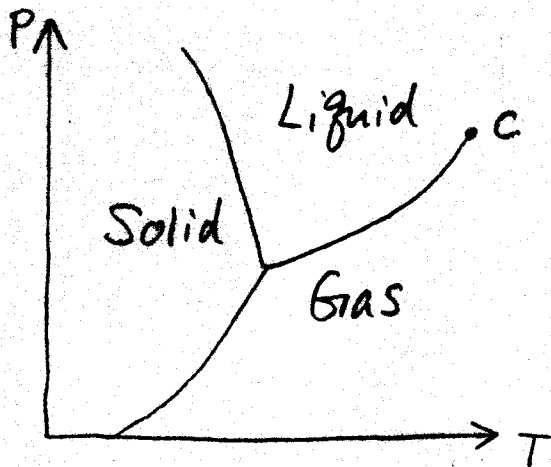
5) (E&M) A conducting sphere of radius R is charged to potential V and is rotated about a diameter at an angular velocity ω . A uniform magnetic field B is applied at right angles to the rotation axis. Find the torque on the sphere.

6) (Stat. Mech./Thermo.) Consider a single-component system. Suppose that the equation of state for the substance is given by

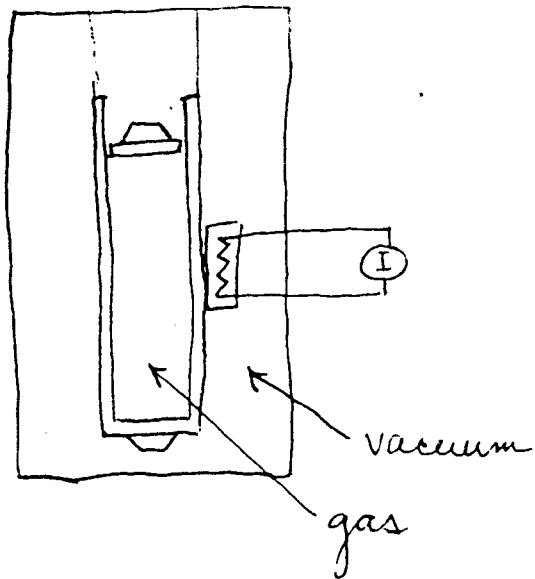
$$\left(p + \frac{a}{V^2}\right)(V - b) = nRT,$$

and that the phase diagram for the substance is shown in the figure.

- Sketch the isotherm-curves for the equation of state of the substance in the p - V plane, and indicate the critical point. (Justify your assignment.)
- Calculate the values of p , V , and T (in terms of a , b , n , and R) at the critical point C .



7) (Stat. Mech./Thermo.) A pipe 0.1 m in diameter is filled with oxygen gas at 300 K and suspended with fine wires in a vacuum chamber. The top end of the pipe is sealed with a piston that slides freely without friction. The piston has mounted on it a loudspeaker for generating sound in the gas, and the total mass of the piston plus loudspeaker is 2 kg. In equilibrium the lowest resonant frequency of the pipe is measured to be 90 Hz, as detected by a microphone at the other end of the pipe. If a current of 0.1 amps is passed through a 10 ohm resistor attached to the pipe for 10 s, what will the new resonant frequency be after equilibrium is established again? (After obtaining formulas algebraically, do the arithmetic by hand to a reasonable precision, i.e., enough to get the direction of change and the approximate size of the relative change. The gas constant R is 8.31 J/(mole-K).)



COMPREHENSIVE WRITTEN EXAMINATION FOR THE MASTER'S DEGREE
AND QUALIFYING EXAMINATION FOR THE PH.D. DEGREE
DEPARTMENT OF PHYSICS
Thursday, April 2, and Friday, April 3, 1998

PART II: FRIDAY, APRIL 3

Important — please read carefully.

The exam (6 hours) is in two parts:

- | | |
|---------------|--|
| Part 1 | Electromagnetic Theory, Statistical Mechanics and Thermodynamics |
| April 2 | 7 Problems — DO ALL PROBLEMS. |
| 9:00-12:00 | This part will be collected at the end of three hours.
Each problem counts for 20 points; the total is 140. |
| Part 2 | Quantum Mechanics, Statistical Mechanics and Thermodynamics |
| April 3 | 7 Problems — DO ALL PROBLEMS. |
| 9:00-12:00 | This part will be collected at the end of three hours.
Each problem counts for 20 points; the total is 140. |

Instructions

- 1) This is a **closed book** exam and calculators are not to be used.
- 2) Work each problem on a separate sheet of paper. Use one side only.
- 3) Print your **name and problem number** on **EACH AND EVERY** page. (Note: Pages without names may not be counted.)
- 4) Return the problem page as the first page of your answer.
- 5) If a part of any question seems ambiguous to you, state clearly what your interpretation is and answer the question accordingly.

8) (Quantum Mechanics) In considering the hydrogen atom, one first solves the spinless non-relativistic potential problem for the Coulomb potential. The spacing of the lowest energy levels is of order $\alpha^k (mc^2)$, where α is the fine structure constant and mc^2 is the rest energy of the electron (actually of the reduced mass). What is the value of the integer k ? (No derivation needed.)

One then considers a number of corrections: a) spin-orbit, b) relativistic correction to kinetic energy, c) effect of external magnetic field, d) spin-spin interaction of the electron and nuclear spins, e) Lamb shift. Without going into an excessive amount of math, describe (without mathematical derivation) the physical origin of each of these corrections, their relative size (power of α where appropriate, or other scaling where appropriate), and their effect on splitting of the $n=1$ and $n=2$ levels of hydrogen. Be precise about what quantum numbers are best used to describe the states.

9) (Quantum Mechanics) Consider two particles, one spin- $\frac{1}{2}$ with spin \vec{S}_1 , and one spin- $\frac{1}{2}$ with spin \vec{S}_2 . They interact with a spin-spin interaction given by

$$H = \frac{4A}{\hbar^2} \vec{S}_1 \cdot \vec{S}_2,$$

where A is a constant.

- Find the energy eigenvalues and the degeneracy of each level.
- Suppose the Hamiltonian is modified by adding a term

$$\frac{2\lambda}{\hbar} S_{2z}$$

where λ is a constant and S_{2z} is the z -component of \vec{S}_2 . How does this change the energy levels and their degeneracy? Be quantitative (and exact, not perturbative).

10) (Quantum Mechanics) Consider a free electron in a static uniform magnetic field given by

$$\vec{B} = (B \sin \theta, 0, B \cos \theta).$$

Neglect the energy due to translational motion. At $t=0$, a measurement of the z component of the electron's spin is made, and the value $+\hbar/2$ is obtained. Calculate the probability $P(t)$ that a measurement of the z component of the electron's spin at time $t \geq 0$ will yield the value $+\hbar/2$. Examine your result in the special cases $\theta=0$ and $\theta=\frac{\pi}{2}$, and comment.

11) (Quantum Mechanics)

a) An electron of charge e is confined to a circular ring of radius R . The plane of the ring lies in the x - y plane, perpendicular to the z -axis, with the origin at the center of the ring. Write the classical Hamiltonian for the electron for this problem in cylindrical coordinates (z , ϕ , and $\rho = \sqrt{x^2 + y^2}$) and their canonical momenta. Then treat this expression as a quantum mechanical Hamiltonian. What are the allowed energy levels?

b) Now let there be an external magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$, constant in time and space. Show that $\vec{A} = \frac{1}{2} \vec{B} \times \vec{r}$. Take \vec{B} in the z -direction: $\vec{B} = B_0 \hat{n}_z$. The Hamiltonian is

$$H = \frac{1}{2m} \left(\vec{p} + \frac{e}{c} \vec{A} \right)^2.$$

What are the quantum energy levels now?

12) (Quantum Mechanics) A particle in 3-D is subject to a potential $V(\vec{r})$ which is non-zero only in a spherical shell of radius a about the origin:

$$V(\vec{r}) = -V_0 a \delta(r - a),$$

where V_0 and a are positive numbers, and δ is the Dirac delta-function. Show which of the following statements are true and which are false:

- There are no values of V_0 and a for which there is a bound state.
- There is a bound state no matter what the values of V_0 and a are.
- The existence of a bound state depends on the values of V_0 and a . (If you say this is true, then give the condition on V_0 and a in order for a bound state to exist.)

13) (Quantum Mechanics) A neutron in 3D (mass M) scatters off a very heavy nucleus, and the force between them is given by the Yukawa potential:

$$V(r) = V_0 \frac{e^{-\mu r}}{\mu r}.$$

What is the Born approximation for the differential cross section, as a function of the scattering angle θ and the neutron's initial momentum $\hbar k$?

14) (Stat. Mech./Thermo.) A system consists of two particles, each of which can occupy any of three quantum states of energy 0 , ϵ , and 3ϵ . The system is in contact with a thermal reservoir at temperature T . What is the mean number of particles in the state with energy ϵ , for the three cases:

- the particles are distinguishable and obey Maxwell-Boltzmann statistics?
- the particles obey Bose-Einstein statistics?
- the particles obey Fermi-Dirac statistics?