COMPREHENSIVE WRITTEN EXAMINATION FOR THE MASTER'S DEGREE AND QUALIFYING EXAMINATION FOR THE PH.D. DEGREE
DEPARTMENT OF PHYSICS
Thursday, April 1, and Friday, April 2, 199~~8~~9

PART I: THURSDAY, APRIL 1

Important — please read carefully.

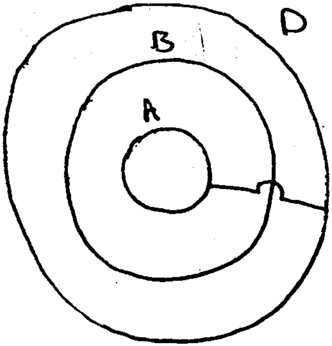
The exam (6 hours) is in two parts:

Part 1	Electromagnetic Theory, Statistical Mechanics and Thermodynamics
April 1	7 Problems — **DO ALL PROBLEMS.**
9:00-12:00	This part will be collected at the end of three hours.
	Each problem counts for 20 points; the total is 140.
Part 2	Quantum Mechanics, Statistical Mechanics and Thermodynamics
April 2	7 Problems — **DO ALL PROBLEMS.**
9:00-12:00	This part will be collected at the end of three hours.
	Each problem counts for 20 points; the total is 140.

Instructions

1) This is a **closed book** exam and calculators are not to be used.
2) Work each problem on a separate sheet of paper. Use one side only.
3) Print your **name and problem number on EACH AND EVERY** page. (Note: Pages without names may not be counted.)
4) Return the problem page as the first page of your answer.
5) If a part of any question seems ambiguous to you, state clearly what your interpretation is and answer the question accordingly.

1) (E&M) A capacitor is made of three thin concentric conducting spherical shells A, B, and D, with radii R_A, R_B, and R_D, respectively, as shown. Spheres A and D are connected by a fine insulated wire passing through a tiny hole in sphere B.



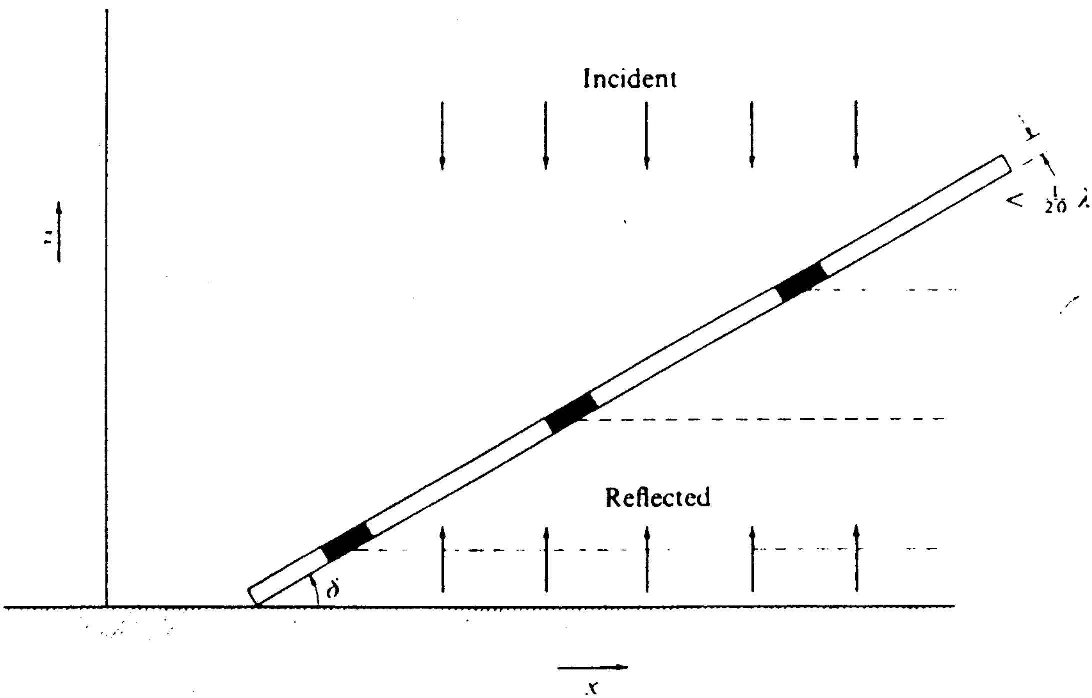
a) Find the capacitance of the system.

b) Suppose a charge, $-Q_B$ is placed on sphere B, and a net charge $+Q_B$ is placed on the sphere A/D system. How does the net charge $+Q_B$ distribute itself between spheres A and D?

2) (E&M) *Wiener's Experiment, as told by J.B. Marion.* In 1889, O. Wiener performed an experiment to investigate whether optical effects are caused predominantly by the electric vector $\vec{E}$ or the magnetic vector $\vec{B}$ of the electromagnetic wave. In his experiment, monochromatic light of wavelength λ is incident on a highly reflecting conducting surface, as in the figure. (You may take the reflection coefficient of the reflecting surface to be unity.) A detector, in the form of a thin ($< \lambda/20$) transparent photographic film, is placed above the reflecting surface, making a small angle δ with the surface.

For definiteness, let the incident EM radiation be plane-polarized in the $\hat{x}$ direction. Then the photographic film is observed to be blackened at regular intervals, as shown. What conclusion did Wiener make?

Your answer should include a rigorous derivation, including a derivation of the formula for the spacing of the intervals in x. Sketch the incident and reflected waves, and explain qualitatively how the directions of reflected $\vec{E}$ and $\vec{B}$ are consistent with tangential-component boundary conditions demanded by Maxwell's equations.



3) (E&M) *Motion of a Charge near a Conductor.*

a) Consider a charge q that sits at rest, at a point $(x_0, 0, 0)$ near a conducting surface – the plane $x = 0$. Calculate the force on the charge.

b) Calculate the time T for the charge to hit the conductor. Ignore any relativistic effects.

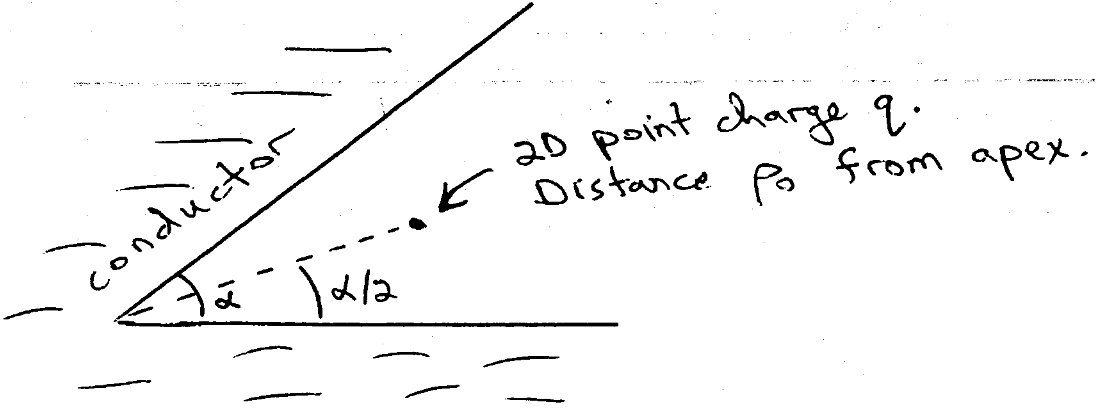
c) Now consider the motion of the charge when there is also a magnetic field $\vec{B} = B_0 \hat{z}$, where B_0 is a constant and $\hat{z}$ is the unit vector in the z direction. In the limit that B_0 is very large, calculate and sketch the motion of the charge. (If you cannot calculate it, still try to sketch it for partial credit.)

4) (E&M) *Electric Dipole in a Spherical Cavity.*

a) Consider two point charges, q at $\vec{r} = (0, 0, d/2)$, and $-q$ at $\vec{r} = (0, 0, -d/2)$. Calculate the electric field of these charges for $r \gg d$. Write your answers in spherical polar coordinates and denote $qd = p$, the dipole strength.

b) Suppose we let $d \to 0$, but p stays finite so that we have an idealized dipole. Now place this dipole at the center of a spherical vacuum cavity of radius r_0, which is inside a dielectric of dielectric constant ϵ. Calculate the electric field inside the cavity and in the dielectric.

5) (E&M) Consider the *two* dimensional configuration shown below.



Derive an expression for the electrostatic potential in the wedge-shaped region bounded by the conductor. Define the potential to be zero on the conductor. (Hint: as far as we know, image charges are *not* a productive way to approach this problem.)

6) (Stat. Mech./Thermo.) Liquid helium boils at a temperature $T_0 = 4.2$ K when its vapor pressure is $P_0 = 1$ atm. The latent heat of vaporization is about $L = 90$ joules/mole and can be taken to be temperature-independent. A few liters of the liquid is held in an insulating dewar, but with a heat influx to the liquid of $dQ/dt = 1$ watt. A mechanical pump is used to lower the temperature of the liquid below T_0. The pump can remove a volume of gas $dV/dt = 200$ liters/s at room temperature (300 K), independent of the gas pressure.

a) Find the vapor pressure of the liquid helium at temperature T, in terms of the latent heat L, P_0, T_0, and numerical and physical constants.

b) What is the minimum temperature of the liquid achieved by the pump?

7) (Stat. Mech./Thermo.) Two identical gases of the same pressure, P, and containing the same number of particles, N, but at different temperatures T_1 and T_2, are in vessels with volumes V_1 and V_2. The vessels are then connected. What is the change in entropy of this system?

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PART II: FRIDAY, APRIL 2

Important — please read carefully.

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Part 1	Electromagnetic Theory, Statistical Mechanics and Thermodynamics
April 1	7 Problems — **DO ALL PROBLEMS.**
9:00-12:00	This part will be collected at the end of three hours.
	Each problem counts for 20 points; the total is 140.
Part 2	Quantum Mechanics, Statistical Mechanics and Thermodynamics
April 2	7 Problems — **DO ALL PROBLEMS.**
9:00-12:00	This part will be collected at the end of three hours.
	Each problem counts for 20 points; the total is 140.

Instructions

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8) (Quantum Mechanics) A particle moves in a potential that looks like the harmonic oscillator for positive x, but is infinite for negative x, so that the wave function must vanish for $x \leq 0$:

$$V(x) = \begin{cases} \infty, & x \leq 0 \\ \frac{1}{2} m\omega^2 x^2, & x > 0 \end{cases}$$

a) Estimate the energy using the variational method with a trial function of the form

$$\psi(x) = \begin{cases} 0, & x \leq 0 \\ Nxe^{-\mu x}, & x > 0 \end{cases}$$

b) What is the exact energy of the ground state of this system?

Hint: In this problem you may want to use the integral

$$\int_o^\infty y^n e^{-ay} dy = \frac{n!}{a^{n+1}}$$

9) (Quantum Mechanics) *Hyperfine structure of hydrogen ground state in external magnetic field, after G.L. Squires.* Denote the z component of the Pauli spin matrix for the electron by σ_{ez} and its eigenstates by α_e and β_e. I.e., the spin of the electron is represented by $\frac{\hbar}{2}\vec{\sigma}_e$, where the vector components of $\vec{\sigma}_e$ are the Pauli spin matrices. Use corresponding notation for the proton nucleus. Consider the following Hamiltonian, which contains only the relevant magnetic terms for this problem:

$$H = B(\mu_e \sigma_{ez} + \mu_p \sigma_{pz}) + W \vec{\sigma}_e \cdot \vec{\sigma}_p,$$

where W is a constant. The first term represents the interaction of the magnetic dipole moments $\vec{\mu}_e$ and $\vec{\mu}_p$ of the electron and proton with an assumed uniform magnetic field B in the z direction; the second term represents the magnetic interaction between the dipoles.

a) Explain why the term with μ_p can be neglected (to roughly what level of approximation)? Then neglect it for all remaining parts of this problem.
b) Write down the eigenstates and eigenvalues of H when $B = 0$.
c) Write down the eigenstates and eigenvalues of H in the limit where B is so large that the third term can be neglected.
d) (most of the credit) Find the eigenvalues of H for general B, and sketch them, connecting the limiting behavior of a) and b).

10) (Quantum Mechanics) *Helium atom: in this problem, do not forget that the electron is spin-$\frac{1}{2}$. Also, you can assign a symbol to any integral of a simple product of functions, and use that symbol without evaluating the integral.*

a) Write down a zeroth-order non-relativistic approximation for the ground state wave function and energy of the Helium atom, corresponding to the following terms only in the Hamiltonian: kinetic energies of the two electrons, and the Coulomb interaction of of the nucleus with each electron. (I.e., neglect the Coulomb interaction between the two electrons, treat the nucleus as point-like and infinitely massive, neglect all spin-dependent forces, etc.) Do the same for the first excited state. Discuss degeneracies.

b) Now add the Coulomb interaction between the two electrons, using 1st-order perturbation theory. Discuss how the levels move and the degeneracies change. For simplicity, you need answer this part only for the states with no orbital angular momentum for either electron; they can be considered separately from the $\ell \neq 0$ states without worry.

11) (Quantum Mechanics) An electron is at rest in a constant magnetic field $\vec{B}$ pointing along the z direction. The Hamiltonian is

$$H = -\vec{\mu} \cdot \vec{B} = g\mu_0 \frac{\vec{S}}{\hbar} \cdot \vec{B},$$

where $\vec{B} = B_0 \hat{z}$, and $\hat{z}$ is the unit vector in the z direction. Since the electron is at rest, you can treat this as a two-state system. Let $|\psi_\pm\rangle$ be the eigenstates of S_z with eigenvalues $\pm\frac{\hbar}{2}$ respectively.

a) What are the eigenstates of the Hamiltonian, and what is the energy difference between them?

b) At time $t = 0$ the electron is in an eigenstate of S_x with eigenvalue $+\hbar/2$. Calculate $|\psi(t)\rangle$ for any t in terms of the constants and $|\psi_\pm\rangle$.

c) For the state you calculated in part (b), what are the expectation values of the three components of the spin at any time t?

12) (Quantum Mechanics) Consider the one dimensional harmonic oscillator. The Hamiltonian is

$$H_0 = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}.$$

a) Use the raising and lowering operators to work out a *first order* differential equation for the ground state wave function.

b) Solve this equation to find the (unnormalized) ground state wave function.

c) Use a similar technique (no second order differential equations allowed!) to work out the wave function for the first excited state.

d) Define the operator

$$U = e^{-ipb/\hbar}$$

for some real number b. Here p is the momentum. Find the ground state wave function (up to the normalization constant) for the Hamiltonian

$$H = U H_0 U^{\dagger}.$$

(Hint: you do not need to show a derivation; just give a justification.)

13) (Quantum Mechanics) A beam of particles scatters off an impenetrable sphere of radius a. I.e., the potential is zero outside the sphere, and infinite inside. The wave function must therefore vanish at $r = a$.

What is the total cross section in the limit of zero incident kinetic energy?

14) (Stat. Mech./Thermo.) The rotational motion of a diatomic molecule is specified by two angular variables, ϕ and θ; by the corresponding canonical conjugate momenta p_ϕ, p_θ; and by the moment of inertia I.

a) What is the kinetic energy of the rotational motion ?

b) Using classical statistics, obtain the rotational partition function in terms of I and temperature T.

c) Calculate the corresponding entropy and specific heat.