COMPREHENSIVE WRITTEN EXAMINATION FOR THE MASTER'S DEGREE AND QUALIFYING EXAMINATION FOR THE PH.D. DEGREE
DEPARTMENT OF PHYSICS
Thursday, September 21, and Friday, September 22, 2000

PART I - THURSDAY, SEPTEMBER 21

Important -- please read carefully.

The exam (8 hours) is in two parts:

Part 1 Electromagnetic Theory, Statistical Mechanics and Thermodynamics

September 21 7 Problems -- **DO ALL PROBLEMS.**

9:00-1:00 This part will be collected at the end of four hours.
Each problem counts for 20 points; the total is 140.

Part 2 Quantum Mechanics, Statistical Mechanics and Thermodynamics

September 22 7 Problems -- **DO ALL PROBLEMS.**

9:00-1:00 This part will be collected at the end of four hours.
Each problem counts for 20 points; the total is 140.

Instructions

1) This is a closed book exam and calculators are not to be used.

2) Work each problem on a separate sheet of paper. Use one side only.

3) Print your name and problem number on EACH **AND EVERY** page. (Note: Pages without names may not be counted.)

4) Return the problem page as the first page of your answer.

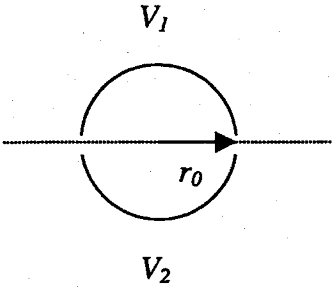
5) If a part of any question seems ambiguous to you, state clearly what your interpretation is and answer the question accordingly.

1. (E&M) The dielectric constant of a insulating material such as sand can be represented by a complex number

$$\varepsilon = \varepsilon' - i\varepsilon'',$$

where ε' and ε'' are both real and $\tan\delta \equiv |\varepsilon''/\varepsilon'| << 1$. For frequencies below atomic resonances, ε is frequency-independent. What is the approximate attenuation length (the distance for power to drop to $1/e$ of its original intensity) of a 1 GHz microwave signal that is propagating through dry sand for which $\tan\delta = 0.02$ and the real part of its index of refraction is 1.6 at 1 GHz.

2. (E&M) Consider a conducting cylinder of radius r_0 which is split along its axis. Half is held at voltage V_1, the other half is held at voltage V_2. What is the electrostatic potential as a function of position outside the cylinder?



3. (E&M) An electromagnetic wave propagates in a dispersive medium of refractive index $n = a^{1/2}/[\omega(b-\omega)]^{1/2}$ where a, b are constants. Plot the dispersion relation $\omega(k)$. Find the maximum group velocity.

4. (E&M) A plane electromagnetic wave in a homogeneous medium ($\varepsilon \neq \varepsilon_0$, $\mu = \mu_0 = 4\pi x 10^{-7}$ Vs/Am) has the following characteristics:

(i) The phase velocities in x,y,z directions are $v_x = \omega/k_x = c$, $v_y = 2c$, $v_z = 3c$.
(ii) The electric field is in the x-y plane and has a field strength $|\mathbf{E}|$=1V/m.

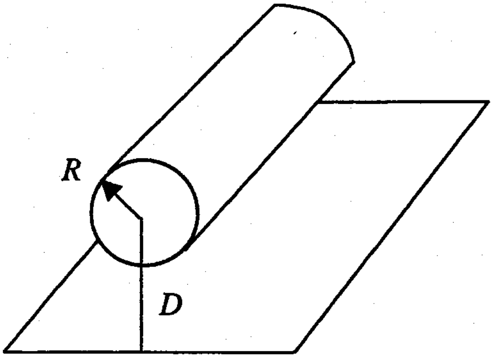
From these facts determine quantitatively:

1) The direction and magnitude of the group velocity, v_g.
2) The direction and magnitude of the magnetic field, $\boldsymbol{H}$.
3) The power flow through the x-y plane, S_z.
4) The wave energy density, u_{em}.

5. (E&M) Consider a capacitor built out of an infinite conducting plane and an infinite conducting cylinder. The radius of the cylinder is $R>0$, its axis is parallel to the plane and the distance between the axis of the cylinder and the plane is $D>R$.

(a) Construct a set-up of 2 conducting lines whose equipotential surfaces have precisely the geometry described above.

(b) Compute the capacitance of the system.



6. (Stat. Mech./Thermo) Consider the Ising model in a magnetic field in one dimension:

$$H = -J\sum_i \sigma_i \sigma_{i+1} - h\sum_i \sigma_i$$

where $\sigma_i = \pm 1$ and J and h are constants.

a) Write an expression for the partition function of the system, in terms of T, J, h, in the limit that the number of sites, $N \to \infty$.

b) What is the magnetization of the system?

7. (Stat. Mech./Thermo) The magnetization, M of a paramagnetic system has the following dependence on the applied magnetic field, H

$$M = N\alpha(H/T)$$

where N is the number of moments in the paramagnet, T is the absolute temperature and α is a constant. The heat capacity at $H = M = 0$ is given by

$$C_H(N, T) = \beta N T^3$$

where β is another constant. Use the above information to construct the magnetic Gibbs free energy, $G(N,T,H)$. From your result derive an expression for the entropy as a function of T and H. Does this system obey the third law of thermodynamics?

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PART II - FRIDAY, SEPTEMBER 22

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Part 2 Quantum Mechanics, Statistical Mechanics and Thermodynamics

September 22 7 Problems -- **DO ALL PROBLEMS.**

9:00-1:00 This part will be collected at the end of four hours.
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8. (Quantum Mechanics) A particle of mass m is constrained to move between two concentric impenetrable spheres of radii $r = a$ and $r = b > a$. There is no other potential. Find the ground state energy and normalized wave function.

9. (Quantum Mechanics) *The Stark effect in hydrogen.*

(a) Write down the Schroedinger equation for the (non relativistic) hydrogen atom, in the presence of an external electromagnetic field. Specialize to the case when the external field is purely electric, constant in time and direction, and uniform in strength.

(b) To deal with this problem by perturbation theory, begin by identifying the unperturbed Hamiltonian H_0 and the perturbation V for this problem. Suppose that the unperturbed atom is in the ground state. Prove that, to first order in the perturbation, there is no energy shift due to the external field.

(c) Your proof may at first seem to apply to the case of an arbitrary unperturbed state, but this is not true because the unperturbed system is degenerate. Consider the first 4 excited states of the atom, all with the same energy, and use degenerate perturbation theory to calculate the splitting. Do not worry about the exact numbers, but explain the structure of the deformed spectrum.

10. (Quantum Mechanics) *Three spins:* A system of three non-identical spin one-half particles, whose operators are s_1, s_2, and s_3, is governed by the Hamiltonian:

$$H = A s_1 \cdot s_2/\hbar^2 + B(s_1 + s_2) \cdot s_3/\hbar^2.$$

Find the energy levels and their degeneracies.

11. (Quantum Mechanics) Consider an electron in a uniform magnetic field in the positive z-direction. The result of a measurement has shown that the electron spin is along the positive x-direction at $t = 0$. Identify the eigenstate and find corresponding eigenvalue of the electron at (a) $t = T/4$, (b) $t = T/2$, and (c) $t = 3T/4$ where $T = 2\pi m/eB$ is the cyclotron period.

12. (Quantum Mechanics) *Many body system obeying Bose-Einstein statistics.*

Suppose given a set of "oscillators": creation operators $a_1^*, a_2^*, \ldots$ and annihilation operators $a_1, a_2, \ldots$, obeying the commutation relations

$$[a_i, a_j^*] = \delta_{ij}.$$

Of course, we know that $[a_i, a_j]$ and $[a_i^*, a_j^*]$ both vanish, and that this implies the symmetry of the states

$$a_i^* a_j^* \ldots |0\rangle,$$

where $|0\rangle$ is the vacuum that satisfies $a_i|0\rangle = 0$. But suppose you are looking for something new, and you propose to give up the commutativity of the a_i^*'s among themselves, and of the a_i's among themselves. This would allow to construct antisymmetric states, as for example the state

$$(a_1^* a_2^* - a_2^* a_1^*)|0\rangle.$$

Prove that this state, under the stated assumptions, has zero norm.

13. (Stat. Mech./Thermo) Consider a system of non-interacting, non-relativistic bosons of mass m in two dimensions. Suppose the density of the bosons is n. What is the asymptotic form of the chemical potential μ in the limit $T \to 0$. Express your answer in terms of T, m, n, $\hbar$.

14. (Stat. Mech./Thermo) Two identical bodies, each characterized by a heat capacity at constant pressure C which is independent of temperature, are used as heat reservoirs for a heat engine. The bodies remain at constant pressure and undergo no change of phase. Initially, their temperature are T_1 and T_2 respectively; finally, as a result of the operation of the heat engine, the bodies will attain a common final temperature T_f

(a) What is the total amount of work W done by the engine? Express the answer in terms of C, T_1, T_2, and T_f.

(b) Use arguments based upon entropy considerations to derive an inequality relating T_f to the initial temperature T_1 and T_2.

(c) For given initial temperatures T_1 and T_2; What is the maximum amount of work obtainable from the engine?