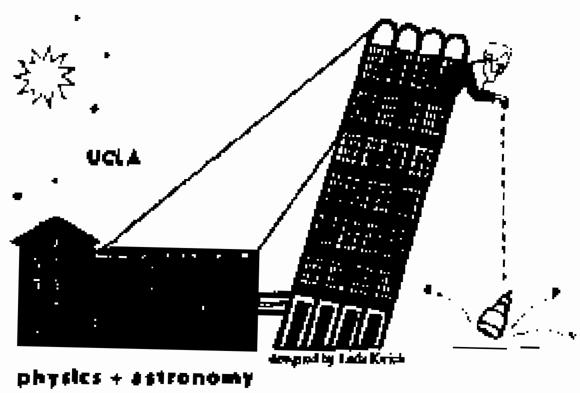
Fall 2007
Physics Comprehensive Exam

September 20, 2007 (Part 1) 9:00 - 1:00pm

Part 1: Quantum Mechanics and Statistical Mechanics
7 Total Problems/20 Points Each/Total 140 Points

- Closed book exam.
- Calculators not allowed.
- Use paper provided for each problem. Use one side only.
- Print your name and page number on every page.
- Return the problem page as the first page of your answers.
- When submitting, please clip all pages together in problem # order.
- If a part of any problem seems ambiguous to you, state clearly your interpretations and answer the question accordingly.



NAME ______________________

1. *Quantum Mechanics*

Consider a system of N identical spin 1/2 particles, each of mass m. They move in a common external potential $V(r) = \frac{1}{2}m\omega^2 r^2$. Ignore mutual interactions among the particles.

Let $E_{\rm gnd}(N)$ be the ground state energy.

a) Compute $E_{\rm gnd}$ exactly for $N = 10$.

b) Consider the limit $N \to \infty$, where

$$\lim_{N\to\infty} \frac{E_{\rm gnd}(N)}{AN^{\alpha}} = 1$$

Find the constant α.

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2. *Quantum Mechanics*

The electrostatic potential of an atomic nucleus can be approximated by the electrostatic potential of sphere of radius R with a homogeneous charge distribution. In a hydrogen-like atom, using this approximation, the electron sees the potential

$$V_{sphere} = \begin{cases} -\frac{3Ze^2}{2R}(1 - \frac{r^2}{3R^2}) & (r \leq R) \\ -\frac{Ze^2}{r} & (r > R) \end{cases}$$

Treat the deviation from the Coulomb potential due to the finite size of the nucleus as a small perturbation, $\hat{V}$. Compute the energy shifts ΔE to first order due to this perturbation.

Hint: The nuclear radius $R \approx A^{1/3} r_0$ with $r_0 = 1.2$ fm is much smaller than the Bohr radius $a_B = 0.53\mathring{A}$. Therefore you can approximate the wave functions appearing in the integrals by their values at $\vec{r} = 0$. Recall also that $|\Psi_{nlm}(0)|^2 = \frac{1}{\pi}\left(\frac{Z}{na_B}\right)^3 \delta_{l0}\delta_{m0}$.

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3. *Quantum Mechanics*

The electron neutrino $|\nu_e\rangle$ and the muon neutrino $|\nu_\mu\rangle$ are the possible neutrino states produced and detected in some experiment, but they are not necessarily eigenstates of the Hamiltonian. Rather, if the state is known to have momentum p, it is some linear combination of the energy eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$ of the form

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$
$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

where

$$H|\nu_1\rangle = \sqrt{p^2c^2 + m_1^2c^4}\,|\nu_1\rangle$$
$$H|\nu_2\rangle = \sqrt{p^2c^2 + m_2^2c^4}\,|\nu_2\rangle$$

for two possibly different masses m_1 and m_2, and some "mixing angle" θ. If it is known that a neutrino was definitely a ν_μ when it was produced, what is the probability of detecting a ν_e after it has traveled a distance L? Assume that $m_1c \ll p$ and $m_2c \ll p$, so that the neutrinos are moving at almost (or even exactly) the speed of light, [so you can ignore corrections of the order $1 - v/c$ compared to terms of order 1] and state your result to first order in the difference $\Delta m^2 = m_1^2 - m_2^2$.

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4. *Quantum Mechanics*

The amplitude for the absorption of a plane-polarized electromagnetic wave off a one-electron atom is proportional to[1]

$$\langle \psi_f | \hat{\epsilon} \cdot \mathbf{p} e^{i\mathbf{k}\cdot\mathbf{r}} | \psi_i \rangle$$

Here $\mathbf{r}$ is the electron position operator and $\mathbf{p}$ is the electron momentum operator. $|\psi_i\rangle$ and $|\psi_f\rangle$ are, respectively the initial and final electronic states. The vector $\mathbf{k}$ is in the direction of the plane wave, and its magnitude $k = |\mathbf{k}|$ is the wave number. The angular frequency of the wave is $\omega = ck$. $\hat{\epsilon}$ is the polarization of the electromagnetic wave. It is a unit vector perpendicular to $\mathbf{k}$: $\hat{\epsilon} \cdot \hat{\epsilon} = 1$ and $\hat{\epsilon} \cdot \mathbf{k} = 0$. [The components of $\mathbf{k}$ and of $\hat{\epsilon}$ are ordinary numbers, not operators.]

Let $|\psi_i\rangle$ be the $1S$ state of a hydrogen atom, and $|\psi_f\rangle$ be the $2S$ state. Suppose the wavelength of the radiation is long enough so that $ka \ll 1$, where a is the Bohr radius. Then if you compute the amplitude by expanding $e^{i\mathbf{k}\cdot\mathbf{r}}$, your answer will be a power series in ka. What is the first non-vanishing term in this expansion? [Write this term as an integral, or sum of integrals, over hydrogen atom wave functions. Don't take the time here to evaluate these integrals.]

[1] in the dipole approximation

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5. *Quantum Mechanics*

Two spin-half particles are in a state with total spin zero. Let $\hat{n}_a$ and $\hat{n}_b$ be unit vectors in two arbitrary directions. Calculate the expectation value of the *product* of the spin of the first particle along $\hat{n}_a$ and the spin of the second along $\hat{n}_b$. That is, if $\mathbf{s}_a$ and $\mathbf{s}_b$ are the spin operators for the first and second particle, respectively, calculate

$$\langle\psi|\, \mathbf{s}_a \cdot \hat{\mathbf{n}}_a \mathbf{s}_b \cdot \hat{\mathbf{n}}_b \,|\psi\rangle$$

Here $|\psi\rangle$ is the two-particle spin-zero state. In this problem ignore orbital angular momentum.

> Hint: Because the state is spherically symmetric the answer can depend only on the angle between the two directions.

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6. *Statistical Mechanics*

Consider a single particle of mass m in a large box of volume V. The potential is zero except for a small region at the center of the box, such that there is one bound state with energy $-U_0$.

a) What is the partition function when this box is in equilibrium with a reservoir of temperature T?

b) Show that even when $k_BT \ll U_0$ the particle becomes unbound if the volume is large enough. How large does the volume have to be for the particle to become unbound in this limit? (Hint: Consider the particle unbound if its average energy approaches the thermal energy of a free particle.)

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7. *Statistical Mechanics*

Two identical containers of volume V each are filled with ideal gas under normal conditions (that is, at room temperature $T = 300\,\mathrm{K}$ and atmospheric pressure $P = 105\,\mathrm{Pa}$). Initially, there are N atoms in each container. After the two containers are connected, the probability to find the occupational asymmetry of $\Delta N/N = 10^{-9}$ is e^{100} times less than the probability of equal occupations in the two containers, $\Delta N = 0$. Using this information, calculate the volume V of each container.

Fall 2007
Physics Comprehensive Exam

September 21, 2007 (Part 2) 9:00 - 1:00pm

Part 2: Electricity & Magnetism and Statistical Mechanics
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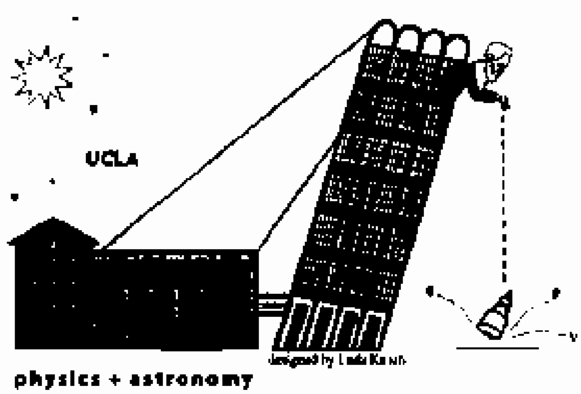

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8. *Electricity and Magnetism*

a) Find the total energy radiated in the collision of a *relativistic* ($\gamma \gg 1$) particle of charge q, mass m, velocity v (in the lab), impact parameter b against a fixed target of charge Q, in the limit of small deflections.

b) Write the condition for "small deflection" in terms of the parameters of the problem.

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9. *Electricity and Magnetism*

The plane $z = 0$ is held at zero potential except for a disk of radius a centered at the origin, which is held at potential V.

a) Write the solution for the potential in the half-space $z \geq 0$. You do not have to explicitly calculate the integral in your expression. (Hint: find the Greens function appropriate for this problem.).

b) Write explicitly the potential on the positive z-axis

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10. *Electricity and Magnetism*

An infinite straight wire is lying along z-axis. At time $t = 0$ the current jumps from zero to a constant value I_0 [i.e. the current is the step function $I(t) = I_0\Theta(t)$.] Calculate, as a function of time, the electric and magnetic fields at an arbitrary point P a distance r from the wire. In particular, consider the cases $r < ct$, $r > ct$, and $t \to \infty$. Don't try to compute the fields at $r = ct$; they are singular here due to the unphysical step function behavior.

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11. *Electricity and Magnetism*

A lossless waveguide has a rectangular cross section of width $2a$ in the x-direction and height a in the y-direction. A plane electromagnetic wave with field components $E_x = E_y = E_o \cos(k_o z - \omega t)$ and free-space wave number $k_o = (3/2)\pi/a$ is incident at the input of the waveguide ($z = 0$). Consider propagation in the fundamental mode, which has a sinusoidal amplitude distribution across the guide normal to the electric field with zero tangential fields at the conducting boundaries. Recall that the wave dispersion is found by separation of variables.

a) What is the polarization of the electric field at $z = 0$? (Neglect reflections.)

b) Consider the propagation of the E_x and E_y fields separately, each propagating in its fundamental waveguide mode. What are the guide wave numbers k_g for each mode?

c) At which distance z is the phase shift between the two fields $\Delta k_g z = \pi/2$? What is the field polarization at this point?

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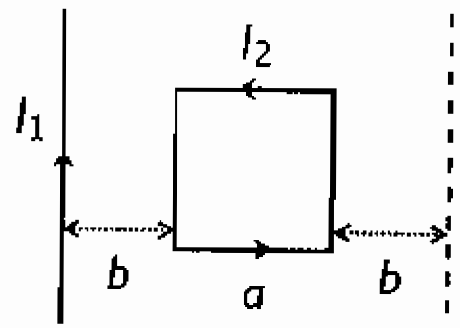
12. *Electricity and Magnetism*



Consider a square loop carrying a constant current I_2 and a long wire with a constant current I_1. The square loop has the area a^2 and the long wire is placed a distance b away from (and parallel to) one side of the square loop, in the same plane as the loop. How much work has to be done in order to slowly move the long wire to the other side of the loop, so that it is finally located the same distance b from the opposite side of the loop?

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13. *Statistical Mechanics*

Consider N rod-like molecules that are arranged in a lattice and have 2 rotational degrees of freedom each associated with the same moment of inertia I. Neglect interactions. Calculate the fluctuation in energy when the molecules are in equilibrium with a heat reservoir at temperature T:

$$\left\langle (\Delta E)^2 \right\rangle = ?$$

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14. *Statistical Mechanics*

Compressing a liquid adiabatically by $\Delta V/V = 10^{-3}$, its temperature rises by one degree, $\Delta T = 1\,\mathrm{K}$. Calculate the ratio of specific heat at constant pressure and constant volume, C_P/C_V, if you know that the constant-pressure thermal expansion coefficient of the liquid is $\beta = (\partial V/\partial T)_P/V = 10^{-4}\,\mathrm{K}^{-1}$.