General useful stuff

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$ ①

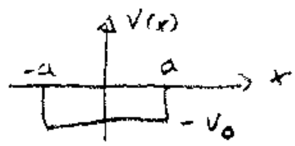
binomial series $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$

Taylor series (about a) $f(a+x) = f(a) + x f'(a) + \frac{x^2}{2!} f''(a) + \cdots$

limits $\left(1 + \frac{x}{n}\right)^n \to e^x$ as $n \to \infty$; $\frac{x^n}{n!} \to 0$ as $n \to \infty$

$x \ln x \to 0$ as $x \to 0$; $n^c x^n \to 0$ as $n \to \infty$ if $|x| < 1$

quantum perturbed eigenvalues: $E_k' = E_k + \langle \psi_k | \hat{H}' | \psi_k \rangle + \sum_{n \neq k} \frac{|\langle \psi_k | \hat{H}' | \psi_n \rangle|^2}{E_k - E_n} + \cdots$

Potential well: 

bound states $V_0 < E < 0$

$$\tan qa = \begin{cases} |k|/q & \text{even parity} \\ -q/|k| & \text{odd parity} \end{cases} \qquad q^2 - |k|^2 = \frac{2m V_0}{\hbar^2}$$

particles in a rectangular box: Energy levels: $E_{lmn} = \frac{h^2}{8M}\left(\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2}\right)$ $l, m, n \geq 1$, M = mass

Thermo:

1st law $dU = đQ + đW$

dU: change in internal energy; $đQ$: heat supplied to system; $đW$: work done on system

2nd law $dS \geq \frac{đQ}{T}$

Thermodynamic work: $đW = -p\,dV = \gamma\, dA = \vec{E} \cdot d\vec{p} = \vec{B} \cdot d\vec{m} = \Delta\phi\, dq$

γ: surface tension; $\vec{p}$: induced electric dipole moment; $\vec{m}$: magnetic dipole moment; $\Delta\phi$: potential difference; dq: charge moved

Cycle coefficients:

Heat engine: $\eta = \frac{\text{work extracted}}{\text{heat input}} \leq \frac{T_h - T_c}{T_h}$

Refrigeration: $\eta = \frac{\text{heat extracted}}{\text{work done}} \leq \frac{T_c}{T_h - T_c}$

heat pump: $\eta = \frac{\text{heat supplied}}{\text{work done}} \leq \frac{T_h}{T_h - T_c}$

Heat capacities:

$$C_V = \left(\frac{đQ}{dT}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V \qquad \gamma = \frac{C_p}{C_V}$$

$$C_p = \left(\frac{đQ}{dT}\right)_p = T\left(\frac{\partial S}{\partial T}\right)_p \qquad ; \qquad C_p - C_V = \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{\partial V}{\partial T}\right)_p$$

Thermodynamic potentials:

$$dU = TdS - pdV + \mu dN$$

enthalpy : $H = U + pV$ $\quad dH = TdS + Vdp + \mu dN$

Helmoltz free energy : $F = U - TS$ $\quad dF = -SdT - pdV + \mu dN$

Gibbs free energy : $G = U - TS + pV$ $\quad dG = -SdT + Vdp + \mu dN$

Maxwell's relations

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V = \frac{\partial^2 U}{\partial S \partial V} \; ; \; \left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p = \frac{\partial^2 H}{\partial p \partial S}$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T = \frac{\partial^2 F}{\partial T \partial V} \; ; \; \left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T = \frac{\partial^2 G}{\partial p \partial T}$$

Latent heat : $L = T(S_2 - S_1)$

Clausius-Clapeyron eq. $\quad \frac{dp}{dT} = \frac{S_2 - S_1}{V_2 - V_1} = \frac{L}{T(V_2 - V_1)}$

Ideal gas

Adiabatic equations : $pV^{\gamma} = \text{const}$, $\quad TV^{(\gamma-1)} = \text{const}$, $\quad T^{\gamma} p^{(1-\gamma)} = \text{const}$

internal energy : $U = \frac{nRT}{\gamma - 1}$

virial coeff

Virial expansion : $pV = RT\left(1 + \frac{B_2(T)}{V} + \frac{B_3(T)}{V^2}\right)$

Monatomic gas : $U = \frac{3}{2} NkT$, $\quad$ pressure $= \frac{1}{3} nm \langle c^2 \rangle$ mean squared particle velocity

$$C_V = \frac{3}{2} Nk, \quad C_p = C_V + Nk = \frac{5}{2} Nk, \quad \gamma = \frac{5}{3}$$

$$S = Nk \ln\left[\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{5/2} \frac{V}{N}\right]$$

Maxwell - Boltzmann distribution

particle energy distribution : $pr(E)dE = \frac{2E^{1/2}}{\pi^{1/2}(kT)^{3/2}} e^{-\beta E} dE$

mean speed : $\langle c \rangle = \left(\frac{8kT}{\pi m}\right)^{1/2}$

Gas equipartition

Classical : $\bar{E}_q = \frac{1}{2} kT$

ideal gas heat capacities : $C_V = \frac{1}{2} \underset{\substack{\uparrow \\ \text{\# of degrees of freedom}}}{f} Nk$, $C_p = Nk\left(1 + \frac{f}{2}\right)$, $\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$

Macroscopic thermodynamic variables

$$F = -kT \ln Z$$
$$U = F + TS = -\left(\frac{\partial \ln Z}{\partial \beta}\right)_{V,N}$$
$$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N} = \frac{\partial (kT \ln Z)}{\partial T}\Big|_{V,N}$$
$$p = -\left(\frac{\partial F}{\partial V}\right)_{T,N} = \left(\frac{\partial kT \ln Z}{\partial V}\right)\Big|_{T,N}$$
$$\mu = \left(\frac{\partial F}{\partial N}\right)_{V,T} = -\frac{\partial (kT \ln Z)}{\partial N}\Big|_{V,T}$$

occupation #

$\langle n_i \rangle = \frac{1}{e^{\beta(\epsilon_i - \mu)} - 1}$ Bose - Einstein

$\langle n_i \rangle = \frac{1}{e^{\beta(\epsilon_i - \mu)} + 1}$ Fermi - Dirac

Fermi energy for non-interacting particles : $\epsilon_F = \frac{\hbar^2}{2m}\left(\frac{6\pi^2 \overset{\substack{\text{\# density} \\ \downarrow}}{n}}{\underset{\substack{\uparrow \\ \text{spin degeneracy } (2s+1)}}{g}}\right)^{2/3}$

Bose condensation temperature : $T_c = \frac{2\pi\hbar^2}{mk}\left[\frac{n}{g\,\underbrace{\zeta(3/2)}_{\text{zeta function} \simeq 2.612}}\right]^{2/3}$

Electrons in solids

$$\vec{J} = -n e v_d \quad , \quad v_d = -\frac{e\tau}{m_e} E$$

mean time between collisions

applied electric field

mean electron drift velocity

Fermi gas

electron density of states: $$g(E) = \frac{V}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} E^{1/2}$$

$$g(E_F) = \frac{3}{2}\frac{nV}{E_F}$$

$$k_F = \left(3\pi^2 \frac{N}{V}\right)^{1/3} \qquad v_F = \frac{\hbar k_F}{m_e}$$

$$E_F = \frac{\hbar^2 k_F^2}{2m_e} \qquad T_F = \frac{E_F}{k_B}$$

$$\frac{C_V}{\text{electron}} = \frac{\pi^2}{3} g(E_F) k_B^2 T = \frac{\pi^2 k_B^2}{2E_F} T$$

$$U_0 = \frac{3}{5} nV E_F$$

$$M = \frac{3n}{2E_F}\mu_0 \mu_B^2 \vec{H}$$

E & M:

Electric dipole $$E_r = \frac{p\cos\theta}{2\pi\epsilon_0 r^3} \qquad E_\theta = \frac{p\sin\theta}{4\pi\epsilon_0 r^3}$$

Image charges

real charge, $+q$, at a distance	image point	image charge
b from a conducting plane	$-b$	$-q$
b from a conducting sphere (radius a)	a^2/b	$-q\,\frac{a}{b}$
b from a plane dielectric boundary:		
seen from free space	$-b$	$-q\left(\frac{\epsilon_r - 1}{\epsilon_r + 1}\right)$
dielectric	b	$\frac{2q}{\epsilon_r + 1}$

electric dipole moment: $\vec{p} = q\,\vec{a}$ or $\vec{p} \int \vec{r}\,'\rho\, d\tau'$

$$\Phi = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} \quad , \epsilon_r = 1 + \chi_E \quad , \vec{D} = \epsilon\vec{E}$$

magnetic dipole moment: $d\vec{m} = \vec{I}\, ds$, $\vec{m} = \frac{1}{2}\int \vec{r}\,' \times \vec{J}\, d\tau'$

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \quad , \vec{B} = \mu\vec{H} \, , \quad \mu_r = 1 + \chi_H$$

Force

on a current-carrying element in a magnetic field $\quad d\vec{F} = I\, d\vec{l} \times \vec{B}$

on a charge $\quad \vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

on an electric dipole $\quad \vec{F} = (\vec{p}\cdot\nabla)\vec{E}$

on a magnetic dipole $\quad \vec{F} = (\vec{m}\cdot\nabla)\vec{B}$

Torque on an electric dipole $\quad \vec{\tau} = \vec{p}\times\vec{E}$ $\qquad$ energy in an electric field $U_{dip} = -\vec{p}\cdot\vec{E}$

magnetic " $\quad \vec{\tau} = \vec{m}\times\vec{B}$ $\qquad$ magnetic " $U_{dip} = -\vec{m}\cdot\vec{B}$

on current loop $\quad \vec{\tau} = I_L \oint \vec{r}\times(d\vec{l}\times\vec{B})$

Hamiltonian of a charged particle in an EM field: $H = \dfrac{|\vec{p}_m - q\vec{A}|^2}{2m} + q\,\phi$

LCR circuits

$$I = \frac{dQ}{dt} \, , V = IR \, , \vec{J} = \sigma\vec{E} \, , \rho = \frac{1}{\sigma} = \frac{RA}{l} \, , C = \frac{Q}{V} \, , I = C\frac{dV}{dt}$$

$$V = -L\frac{dI}{dt} \quad , L_{12} = \frac{\Phi_1}{I_2}$$

energy stored in capacitor: $U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$

inductor: $U = \frac{1}{2}LI^2 = \frac{1}{2}\Phi I = \frac{1}{2}\frac{\Phi^2}{L}$

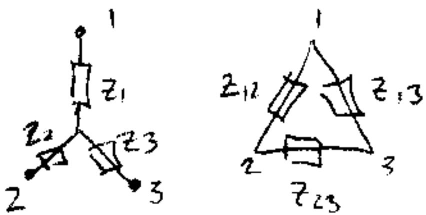
Power dissipated in a resistor: $W = IV = I^2R = \frac{V^2}{R}$

Impedance of capacitance: $Z_C = -\frac{i}{\omega C}$

inductance $Z_L = i\omega L$

Kirchhoff's laws: current law $\sum_{\text{node}} I_i = 0$ Voltage law $\sum_{\text{loop}} V_i = 0$

star-delta transformation



$$Z_{ij} = Z_i Z_j \left(\frac{1}{Z_i} + \frac{1}{Z_j} + \frac{1}{Z_k} \right)$$

$$Z_i = \frac{Z_{ij} Z_{ik}}{Z_{ij} + Z_{ik} + Z_{jk}}$$