#1) Fall 2000 |EM|

The dielectric constant of sand is given as

$$\varepsilon = \varepsilon' - i\varepsilon'' \quad \text{where} \quad \varepsilon', \varepsilon'' \in \mathbb{R}$$

where $\tan\delta = \left|\frac{\varepsilon''}{\varepsilon'}\right| = 0.02$. What is the approximate attenuation length of the power to drop by a factor of $\frac{1}{e}$ for a 1 GHz microwave signal? For sand, the real part of the index of refraction is

$$n_R = 1.6$$

Solution. There are many similar ways to do this problem. It is essentially the skin depth of the POWER of the fields, but we can just recognise a relationship:

$$n = \sqrt{\varepsilon(\omega)} = \sqrt{\varepsilon' - i\varepsilon''} = \sqrt{\varepsilon'}\left(1 - i\frac{\varepsilon''}{\varepsilon'}\right)^{1/2}$$

Since $\frac{\varepsilon''}{\varepsilon'} \ll 1$ $\quad n \approx \sqrt{\varepsilon'}\left(1 - \frac{i}{2}\tan\delta\right)$

For a general wave in the x direction,

$E = E_0 e^{i(kx - \omega t)}$ we have the relation

$$n = \frac{c}{v} = \frac{ck}{\omega} \Rightarrow k = n\frac{\omega}{c} = \frac{\omega}{c}\sqrt{\varepsilon'}\left(1 - \frac{i}{2}\tan\delta\right)$$

So $E \propto e^{i\left[\frac{\omega}{c}\sqrt{\varepsilon'}\left(1 - \frac{i}{2}\tan\delta\right)x - \omega t\right]}$

$\propto e^{\frac{\omega}{2c}\sqrt{\varepsilon'}\tan(\delta)x}\, e^{i(k_R x - \omega t)}$ this is the oscillatory piece

this gives the attenuation of the field

But since we are looking for the attenuated POWER...

$$P \propto |E|^2 \propto e^{-\frac{\omega}{c}\sqrt{\varepsilon'}\tan\delta\, x}$$

we find when $P_{Att} = \frac{1}{e} P \Rightarrow e^{-\frac{\omega}{c}\sqrt{\varepsilon'}(\tan\delta) d} = \underbrace{e^{-1}}_{\frac{P(x=0)}{e}}$

so

$$\frac{\omega}{c}\sqrt{\varepsilon'}\tan\delta = \frac{1}{d} \leftarrow \text{attenuation depth}$$

$$d = \frac{c}{\omega\sqrt{\varepsilon'}\tan\delta}$$

$$= \frac{c}{2\pi\nu\, n_R \tan\delta}$$

$$= \frac{3\times10^{10}}{2\pi(1e9)(1.6)(0.02)}$$

$$\approx \underline{\underline{150\ \text{cm}}}$$