14) Two identical bodies with heat capacity C
Fall 2000 (independent of temperature) are heat reservoirs for a heat engine. The bodies remain at constant pressure and undergo no change of phase. Initially their temps are T_1 & T_2. As a result of operation of the heat engine, the bodies attain a final temp of T_f.

a) what is the total amount of W done by the engine?

$$\Delta E = -W = +C(T_f - T_1) + C(T_f - T_2)$$

$$-W = C(2T_f - T_1 - T_2)$$

$$\boxed{W = C(T_1 + T_2 - 2T_f)}$$

b) Using arguments based on entropy considerations, derive an inequality relating T_f to T_1 & T_2.
The total change in entropy is ...

$$\Delta S = \Delta S_1 + \Delta S_2 \geq 0$$

$$= C\int_{T_1}^{T_f} \frac{dT}{T} + C\int_{T_2}^{T_f} \frac{dT}{T} \geq 0$$

$$= C\left[\ln\left(\frac{T_f}{T_1}\right) + \ln\left(\frac{T_f}{T_2}\right)\right] \geq 0$$

so ... $\ln\left(\frac{T_f}{T_1}\right) \geq -\ln\left(\frac{T_f}{T_2}\right)$

$$\boxed{T_f \geq \sqrt{T_1 T_2}}$$

c) for T_1 & T_2 initial temperatures, what is maximum W?
If $\Delta S = 0 \Rightarrow W_{max}$. So $T_f = \sqrt{T_1 T_2} \Rightarrow \boxed{W_{max} = C(T_1 + T_2 - 2\sqrt{T_1 T_2})}$