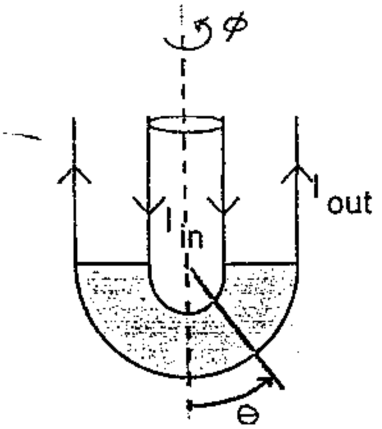

p1

Two concentric metallic electrodes have hemispherical ends. A resistive material fills the space between the concentric hemispheres. A current I flows through the system. Find $\overrightarrow{H}(r,\theta)$ inside the resistive medium.

Start with the Maxwell equation

$$\nabla \times \overrightarrow{H} = \frac{4\pi}{c}\overrightarrow{J_f} + \frac{1}{c}\frac{\partial \overrightarrow{D}}{\partial t} \qquad \text{(Gaussian units)}$$

since $\frac{\partial \overrightarrow{D}}{\partial t} = 0$ for this system, one is left with

$$\nabla \times \overrightarrow{H} = \frac{4\pi}{c}\overrightarrow{J_f}.$$

This equation is equivalent to Ampere's Law, which allows us to write

$$\oint \overrightarrow{H}\cdot\overrightarrow{dl} = \frac{4\pi}{c}\overrightarrow{I_{enc}}$$

Exploit the symmetry of the problem by performing a line integral over a circular loop centered on the axis of the system. The line element in the $\hat{\varphi}$ direction is given by

$$\overrightarrow{dl_\varphi} = r\sin\theta\,\overrightarrow{d\phi}$$

so integration yields

$$\oint \overrightarrow{H}\cdot\overrightarrow{dl} = H(2\pi r\sin\theta) = \frac{4\pi}{c}I_{enc}$$

Now find I_{enc} by considering a hemispherical shell of radius r located within the resistive material which shares a common center with the hemispherical electrodes of the system. The current flux through this imaginary shell is constant and radius independent. Thus, the current density must be given by

$$J = \frac{I}{2\pi r^2}$$

I_{enc} can now be found by integrating over the portion of a shell bound by the line integral on H.

$$I_{enc} = \frac{I}{2\pi r^2}\int_0^{2\pi}\int_0^{\theta} r^2\sin\theta' d\theta' d\phi = I(-\cos\theta')_0^{\theta}$$

$$I_{enc} = I(1-\cos\theta)$$

Combining results yields

$$H(2\pi r\sin\theta) = \frac{4\pi}{c}I(1-\cos\theta)$$

$$H = \frac{2I}{cr}\frac{(1-\cos\theta)}{\sin\theta}$$

From the right hand rule and symmetry considerations, $\overline{H}$ must point in the $-\hat{\phi}$ direction, so

$$\overrightarrow{H}(r,\theta) = \frac{2I(\cos\theta - 1)}{cr\sin\theta}\hat{\phi}$$