A system of 1 mole of ideal gas has an initial volume V_1 and a temperature T_1.

a) When the state of the system is changed to a volume V_2 and temperature T_2, show that the entropy change is:

$$\Delta S = c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right)$$

Start from the fundamental thermodynamic relation:

$$dE = T\,dS - p\,dV \quad \Rightarrow \quad dS = \frac{1}{T}dE + \frac{p}{T}dV$$

The ideal gas equation of state is $PV = NRT$ $(N=1)$

So

$$dS = \frac{1}{T}dE + \frac{R}{V}dV$$

Next find dE for ideal gas, letting $E = E(T,V)$

$$dE = \underbrace{\cancel{\left(\frac{\partial E}{\partial V}\right)_T}}_{=0 \text{ for ideal gas}} dV + \left(\frac{\partial E}{\partial T}\right)_V dT = c_v\,dT$$

Thus

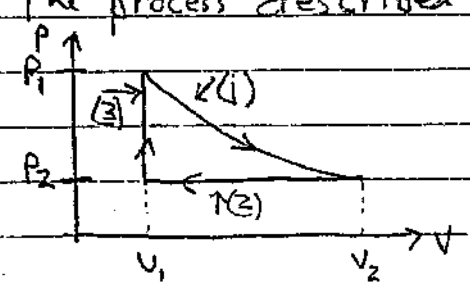
$$dS = \frac{c_v}{T}dT + \frac{R}{V}dV$$

Integrating both sides yields yields

$$\boxed{\Delta S = c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right)}$$

b) Assuming the system undergoes the following process
(1) from (p_1, V_1) to (p_2, V_2) by adiabatic expansion
(2) from (p_2, V_2) to (p_2, V_1) by constant-pressure compression
(3) from (p_2, V_1) to (p_1, V_1) by constant-volume heat absorption
find the efficiency of this process

The process described has the following PV curve:



The efficiency of the process is given by

$$\eta = \frac{W_{out}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}}$$

$\Longrightarrow$

process (1) is adiabatic so $Q_{in} = 0$ $Q_{out} = 0$

in process (2) the volume is being reduced at constant pressure, so there is a Q_{out}

$$Q_{out} = \int dQ = \int dE + \int dW$$

$$\int dE = \int C_v dT = \int_{T_1}^{T_2} (C_p - R) dT \qquad (\text{used } C_v = C_p - R)$$

from eq of state $T = \frac{PV}{R} \Rightarrow dT = \frac{P}{R} dV$

$$\int dE = (C_p - R)\frac{P_2}{R}\left[V_1 - V_2\right]$$

$$\int dW = \int p\,dV = p_2\left[V_1 - V_2\right]$$

So $Q_{out} = \left(\frac{C_p P_2}{R} - \cancel{P_2} + \cancel{P_2}\right)\left[V_1 - V_2\right] = \frac{C_p P_2}{R}\left[V_1 - V_2\right]$

in process (3) volume is constant so $\int dW = 0$, thus

$$Q_{in} = \int dE = \int C_v dT$$

from eq of state $dT = \frac{V}{R} dp$

$$Q_{in} = C_v \frac{V_1}{R}\left[P_1 - P_2\right]$$

Thus $\eta = \frac{Q_{in} - |Q_{out}|}{Q_{in}} = \frac{c_v V_1 (p_1 - p_2) + c_p p_2 (V_1 - V_2)}{c_v V_1 (p_1 - p_2)}$

since $V_1 < V_2$

$$\eta = 1 + \frac{c_p}{c_v}\frac{p_2}{V_1}\frac{(V_1 - V_2)}{(p_1 - p_2)}$$

Rearrangement leads to

$$\boxed{\eta = 1 + \gamma\frac{\left(\frac{V_2}{V_1} - 1\right)}{\left(\frac{p_1}{p_2} - 1\right)}} \qquad \text{where } \gamma = c_p/c_v$$