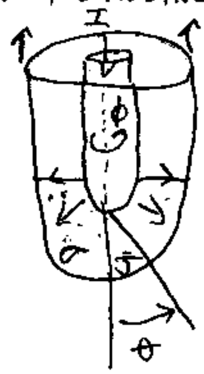
Spring 2000 #1 (p 1 of 1)

Two concentric metallic electrodes have hemispherical ends. A resistive material fills the space between the concentric hemispheres. A current I flows through the inner electrode, through the resistive material and, finally returns through the outer electrode. The system is axially symmetric ($\partial/\partial\phi = 0$). Find the magnetic field inside the resistive material, $\vec{H}(r,\theta)$



Ampère's law in matter

$$c\,\nabla\times\vec{H} = 4\pi\vec{J}_f + \frac{\partial\vec{D}}{\partial t} \qquad , \frac{\partial\vec{D}}{\partial t} = 0 \text{ for this system}$$

So, we have

$$\nabla\times\vec{H} = \frac{4\pi}{c}\vec{J}_f \Rightarrow \oint\vec{H}\cdot d\vec{l} = \frac{4\pi}{c}I_{enc}$$

Taking advantage of the the symmetry in ϕ we have $\vdots$ $d\vec{l}_\phi = r\sin\theta\, d\vec{\phi}$ (see front cover of griffiths)

$$|\vec{H}|\, 2\pi r\sin\theta = \frac{4\pi}{c}I_{enc} \qquad (1)$$

where

$$J = \frac{I}{2\pi r^2} \Rightarrow I_{enc} = \frac{I}{2\pi r^2}\underbrace{\int_0^{2\pi}\int_0^{\theta}}_{\text{just the}} r^2\sin\theta' d\theta' d\phi' = I\,[-\cos\theta]_0^{\theta}$$

just the part of the shell bound by the line integral on H

$$\Rightarrow I_{enc} = I(-\cos\theta + 1) = I(1-\cos\theta) \quad (2)$$

substituting eq (2) into eq (1), we get

$$|\vec{H}|\, 2\pi r\sin\theta = \frac{4\pi I}{c}(1-\cos\theta)$$

$$\Rightarrow |\vec{H}| = \frac{2I}{cr}\frac{(1-\cos\theta)}{\sin\theta}$$

from the right hand rule and symmetry, $\vec{H}$ must point in the $-\hat{\phi}$ direction, so

$$\boxed{\vec{H} = \frac{2I}{cr}\frac{(\cos\theta - 1)}{\sin\theta}\hat{\phi}}$$