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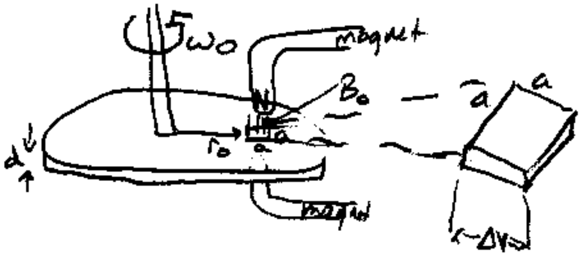
Calculate the torque tending to slow the rotation of the conducting disk shown below. The braking is generated by turning on a small region of uniform magnetic field B_0 over an area $a \times a$ at a distance r_0 from the axis of rotation of the disk, where $a \ll r_0$. The disk is rotating at angular velocity of ω_0 and has a thickness d and a resistivity ρ. Find the torque in terms of the quantities given.



the motion of the current in $\vec{\phi}$ cross the magnetic field in $-\hat{z}$ creates a potential difference in the $\vec{r}$ direction which is the direction of the eddy current that we care about

so,

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad , \text{where } \vec{F} = I \int d\vec{l} \times \vec{B} = I B_0 a (-\hat{\phi}) \quad (1)$$

but note that I depends on r. To find I, use ohm's law. That is,

$$\mathcal{E} = IR \Rightarrow I = \frac{\mathcal{E}}{R} = \frac{|-\int \vec{E} \cdot d\vec{l}|}{\frac{\rho a}{ad}} = \frac{d\,|\vec{E}|\,a}{\rho} = \frac{da}{\rho}\,|\vec{v} \times \vec{B}|$$

$$\therefore \quad I = \frac{da}{\rho} r_0 \omega_0 B_0 \qquad (2)$$

Substitute eq (2) into eq (1) :

$$\vec{F} = -\frac{da^2 B_0^2 r_0 \omega_0}{\rho} \hat{\phi}$$

so,

$$\boxed{\vec{\tau} = -\frac{d a^2 r_0^2 B_0^2 \omega_0}{\rho} \hat{z}}$$