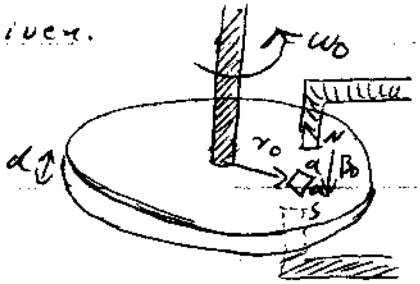
S'00 #4; E.M.

Calculate the torque tending to slow the rotation of the conducting disk shown below. The braking is generated by turning on a small region of uniform magnetic field B_0 over an area $a \times a$ at a distance r_0 from the axis of rotation of the disk, where $a \ll r_0$. The disk is rotating at an angular velocity ω_0 and has a thickness d and a resistivity ρ. Find the torque in terms of the quantities given.



$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{E} = 0$$

$$\vec{J} = \sigma(\vec{v} \times \vec{B}), \quad \vec{v} = \omega_0 r_0 (\hat{\phi}); \quad \vec{B} = B_0(-\hat{z})$$

$$\vec{J} = \sigma \omega_0 r_0 B_0 \big(\underbrace{\hat{\phi} \times (-\hat{z})}_{= -\hat{r}}\big) = \sigma \omega_0 r_0 B_0 (-\hat{r})$$

now $\vec{J} = \dfrac{\vec{I}}{ad} \Rightarrow \vec{I} = \sigma \omega_0 r_0 B_0 ad(-\hat{r}) = \underbrace{\dfrac{\omega_0 r_0 B_0 ad}{\rho}}_{\equiv I_0}(-\hat{r}) = I_0(-\hat{r})$ (as $\sigma = 1/\rho$)

$$\vec{F} = \int_0^a I_0 \, d\vec{\ell} \times \vec{B} = I_0 a B_0 \big[\underbrace{(-\hat{r}) \times (-\hat{z})}_{(-\hat{\phi})}\big] = I_0 a B_0 (-\hat{\phi})$$

$$\vec{\tau} = \vec{r} \times \vec{F} = r_0 I_0 a B_0 \big[\underbrace{\hat{r} \times (-\hat{\phi})}_{(-\hat{z})}\big] = r_0 I_0 a B_0 (-\hat{z})$$

$$= r_0 \frac{\omega_0 r_0 B_0 ad}{\rho} a B_0 (-\hat{z}) = \frac{\omega_0 r_0^2 a^2 B_0^2 d}{\rho}(-\hat{z})$$