Fall 2001 #1

$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 \qquad \hbar = 1 \Rightarrow H|n\rangle = (n+\tfrac{1}{2})\omega |n\rangle$

$\underbrace{\quad}_{E_n}$

$x = \sqrt{\frac{1}{2m\omega}}(a + a^\dagger) \qquad p = \frac{\sqrt{2\omega m}}{2i}(a - a^\dagger)$

a) $\langle n|p^2|n\rangle = -\frac{\omega m}{2}\langle n|(a-a^\dagger)^2|n\rangle = -\frac{\omega m}{2}\langle n|aa + a^\dagger a^\dagger - aa^\dagger - a^\dagger a|n\rangle$

$H = \frac{\omega}{2}(a^\dagger a + aa^\dagger)$

$= \frac{\omega m}{2}\langle n|a^\dagger a + aa^\dagger|n\rangle = \frac{\omega m}{2}\frac{2}{\omega}\langle n|H|n\rangle = m\langle n|H|n\rangle$

$= m(n+\tfrac{1}{2})\omega = m\omega(n+\tfrac{1}{2})$

b) $\langle n|x^2|n\rangle = \frac{1}{2m\omega}\langle n|(a+a^\dagger)^2|n\rangle = \frac{1}{2m\omega}\langle n|aa + aa^\dagger + a^\dagger a + a^\dagger a^\dagger|n\rangle$

would mate $\langle n|n-2\rangle$; $\langle n|n+2\rangle$

$H = \frac{1}{2}\omega(a^\dagger a + aa^\dagger)$ ★

$= \frac{1}{2m\omega}\langle n|aa^\dagger + a^\dagger a|n\rangle = \frac{2}{2m\omega^2}\langle n|H|n\rangle = \frac{1}{m\omega^2}(n+\tfrac{1}{2})\omega$

$= \frac{(n+\frac{1}{2})}{m\omega}$