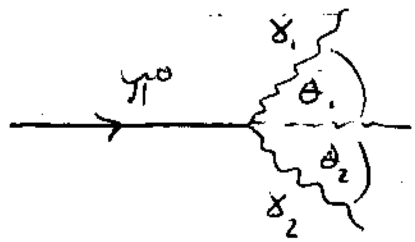
Spring 2001 # 10

A π^0 of velocity v_0 decays in flight into two photons $\pi^0 \to 2\gamma$. Compute the minimum & maximum values of the energies of the produced photons as a function of v_0.

In Lab frame:



- energy conservation: $\gamma m_\pi c^2 = E_1 + E_2$ (I) where 1, 2 refers to γ_1 & γ_2

- momentum conservation: $\gamma m_\pi v_0 = \frac{E_1}{c}\cos\theta_1 + \frac{E_2}{c}\cos\theta_2$ (II)

$$\frac{E_1}{c}\sin\theta_1 = \frac{E_2}{c}\sin\theta_2 \qquad \text{(III)}$$

$$(\text{w/ } m_\gamma = 0 \quad E_\gamma = P_\gamma c)$$

(II): $\left(\gamma m v_0 - \frac{E_1}{c}\cos\theta_1\right)^2 = \left(\frac{E_2}{c}\cos\theta_2\right)^2$

$$= \left(\frac{E_2}{c}\right)^2 - \left(\frac{E_2}{c}\sin\theta_2\right)^2$$

$$\left(\gamma m v_0 - \frac{E_1}{c}\cos\theta_1\right)^2 + \left(\frac{E_1}{c}\sin\theta_1\right)^2 = \left(\frac{E_2}{c}\right)^2$$

used (III)

$$= \left(\gamma m c - \frac{E_1}{c}\right)^2$$

$$\to \gamma^2 m^2 v_0^2 - \frac{2\gamma m v_0 E_1\cos\theta_1}{c} + \cancel{\frac{E_1^2}{c^2}\cos^2\theta_1} + \cancel{\frac{E_1^2\sin^2\theta_1}{c^2}} = \gamma^2 m^2 c^2 - 2\gamma m E_1 + \cancel{E_1^2/c}$$

$$\rightarrow \gamma^2 m^2 (v^2 - c^2) - 2\gamma m E_1 \left(\frac{v_0}{c}\cos\theta_1 - 1\right) = 0$$

$$\Rightarrow E_1 = \frac{\gamma m (v_0^2 - c^2)}{2\left(\frac{v_0}{c}\cos\theta - 1\right)} = \frac{\gamma m c^2 \left(\frac{v_0^2}{c^2} - 1\right)}{2\left(\frac{v_0}{c}\cos\theta - 1\right)}$$

now w/ $\gamma = \left(1 - \frac{v_0^2}{c^2}\right)^{-1/2}$,

$$E_1 = \frac{mc^2}{2\gamma\left(1 - \frac{v_0}{c}\cos\theta_1\right)}$$

- In lab frame $\cos\theta_1$ can range from -1 to 1 corresponding to minimum & maximum energies. Photons can fly backward even in the lab frame because they travel at c and thus cannot be Lorentz boosted to the forward direction when they are travelling very-backward in the CM frame.

$$\rightarrow E_{max} = \frac{mc^2}{2\gamma\left(1 - \frac{v_0}{c}\right)} \quad \& \quad E_{min} = \frac{mc^2}{2\gamma\left(1 + \frac{v_0}{c}\right)}$$

$\rightarrow$ similar expressions can be derived for the min & max energies of the 2nd photon