Problem #14 Spring 2001

a) $F = -KT \ln Z$

$$Z = \frac{1}{N!}\left(\frac{V}{(2\pi\hbar^2/MKT)^{3/2}}\right)^N$$

$$F = -KT\left[N \ln\left(\frac{V}{(2\pi\hbar^2/MKT)^{3/2}}\right) - \ln N!\right]$$

aside

$$Z = \sum_i e^{-\beta\epsilon_i} \xrightarrow{\text{classical}} \int e^{-\beta\frac{p^2}{2m}} \frac{d\vec{p}\, d\vec{r}}{h_0^{3N}}$$

$$F = -KT\left[N \ln V - \tfrac{3}{2} N \ln(2\pi\hbar^2) + \frac{3}{2} N \ln MKT - \ln N!\right]$$

$$F = -NKT\left[\ln V - \frac{3}{2}\ln(2\pi\hbar^2) + \frac{3}{2}\ln(MKT) - \ln N - 1\right]$$

($N \ln N - N$)

b)

$$Z = Z' Z_q = \frac{1}{N!}\left(\frac{V}{(2\pi\hbar^2/mKT)^{3/2}}\right)^N \sum_i e^{-\beta\epsilon_i}$$

where $\epsilon_i = -\frac{\alpha^2 m}{2n^2} = \frac{-me^4}{2\hbar^2 n^2}$

$$\epsilon_i = \frac{-me^4}{8\varepsilon_0^2 h^2 n^2} = \frac{-me^4}{32\pi^2\epsilon_0^2\hbar^2 n^2}$$

$$Z_q = \left(\sum_{n=1}^{\infty} e^{+\beta\frac{\alpha}{n^2}}\right)^N$$

$$F = -NKT\left[\text{Classical Impiece} + \ln\sum_{n=1}^{\infty} e^{+\beta\frac{\alpha}{n^2}}\right]$$