Fall 2002 #1.

$T=0 \quad \begin{pmatrix}0\\1\end{pmatrix} \qquad \hbar=1 \qquad \vec{B}=B_0\hat{x}$

$$H = \frac{g\,\mu_0\,\vec{\sigma}\cdot\vec{B}}{2} = \mu_0 B_0\,\sigma_x \qquad H = \mu_0 B_0 \begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$\lambda = \pm\mu_0 B_0 \quad \frac{1}{\sqrt2}\begin{pmatrix}1\\-1\end{pmatrix},\ \frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix}$

$\mu_0 B_0 = \alpha \qquad |\chi_-\rangle \quad |\chi_+\rangle$

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$
$$= c_1 e^{-iHt}|\chi_+\rangle + c_2 e^{-iHt}|\chi_-\rangle = c_1 e^{-i\alpha t}|\chi_+\rangle + c_2 e^{i\alpha t}|\chi_-\rangle$$

$$|\psi(0)\rangle = \begin{pmatrix}0\\1\end{pmatrix} = c_1\frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix} + \frac{c_2}{\sqrt2}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$0 = \frac{c_1}{\sqrt2} + \frac{c_2}{\sqrt2} \qquad 1 = \frac{c_1}{\sqrt2} - \frac{c_2}{\sqrt2}$$

$c_1 = -c_2$

$$1 = \frac{c_1}{\sqrt2} + \frac{c_1}{\sqrt2} = 1 = \frac{2c_1}{\sqrt2} \qquad c_1 = \frac{\sqrt2}{2} = \frac{1}{\sqrt2}$$

$c_2 = -\frac{1}{\sqrt2}$

$$|\psi(t)\rangle = \frac{1}{\sqrt2}e^{-i\alpha t}|\chi_+\rangle - \frac{1}{\sqrt2}e^{i\alpha t}|\chi_-\rangle$$

$$= \frac{1}{2}\begin{pmatrix} e^{-i\alpha t} - e^{i\alpha t} \\ e^{-i\alpha t} + e^{i\alpha t}\end{pmatrix} = \begin{pmatrix} -i\sin(\omega t) \\ \cos(\omega t)\end{pmatrix} \qquad \omega = \alpha$$

a) $P_{z\uparrow} = |\langle +|\psi(t)\rangle|^2 = \left|(1\ \ 0)\begin{pmatrix}-i\sin(\omega t)\\ \cos(\omega t)\end{pmatrix}\right|^2 = \sin^2(\omega t)$

$P_{z\downarrow} = \cos^2(\omega t)$

b) $P_{x\uparrow} = |\langle \chi_+|\psi(t)\rangle|^2 = \left|\frac{1}{\sqrt2}(1\ \ 1)\begin{pmatrix}-i\sin(\omega t)\\ \cos(\omega t)\end{pmatrix}\right|^2 = \frac{1}{2}$

$P_{x\downarrow} = \frac{1}{2}|-i\sin(\omega t) - \cos(\omega t)|^2 = \frac{1}{2}$