10. *Electricity and Magnetism*

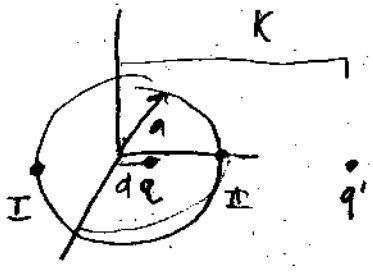
A point charge q is inside a hollow, grounded, conducting sphere of inner radius a. Use the method of images to find

(a) the potential inside the sphere;

(b) the induced surface-charge density at the point on the sphere nearest to q;

(c) the magnitude and direction of the force acting on q.

(d) Is there any change in the solution if the sphere is kept at a fixed potential V? If the sphere has a total charge Q on its inner and outer surface?



$\phi(I, II) = 0$

A+ I

$$\frac{q}{a+d} + \frac{q'}{K+a} = 0 \qquad q' = -\frac{q(K+a)}{(a+d)}$$

II

$$\frac{q}{a-d} + \frac{q'}{K-a} = 0 \qquad q' = -\frac{q(K-a)}{(a-d)} \Rightarrow \frac{(K+a)}{(a+d)} = \frac{(K-a)}{(a-d)}$$

$$\Rightarrow Kd = a^2$$

$$K = \frac{a^2}{d}$$

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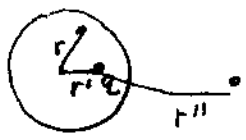
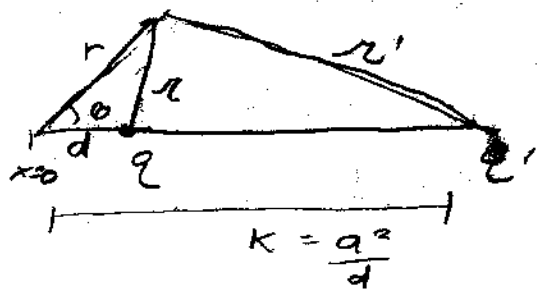
$$q' = -\frac{q\left(\frac{a^2}{d}+a\right)}{(a+d)} = -\frac{qa}{d}\left(\frac{a+d}{a+a}\right) = -\frac{qa}{d}$$

a) $\phi = \frac{q/4\pi\epsilon_0}{|\vec{r}-\vec{r}'|} + \frac{\frac{qa}{d}\frac{1}{4\pi\epsilon_0}}{|\vec{r}-\vec{r}''|}$





$r = r^2 + d^2 - 2rd\cos\theta$

$r' = r^2 + \frac{a^4}{d^2} - 2r\frac{a^2}{d}\cos\theta$

b) $\sigma = -\epsilon_0 \frac{dV}{dn} = -\epsilon_0 \frac{dV}{dr}\Big|_{r=a}$

$$\sigma = -\frac{q}{4\pi}\frac{d}{dr}\left(\frac{1}{r^2+d^2-2rd\cos\theta} - \frac{\frac{q}{d}}{r^2+\left(\frac{a^2}{d}\right)^2 - \frac{2ra^2}{d}\cos\theta}\right)$$

$$= \frac{-q}{4\pi}\left[\frac{-1}{(r^2+d^2-2rd\cos\theta)^2}\cdot(2r-2d\cos\theta) + \frac{\frac{q}{d}}{\left(r^2+\left(\frac{a^2}{d}\right)^2-\frac{2ra^2}{d^2}\cos\theta\right)^2}\left(2r-2\frac{a^2}{d}\cos\theta\right)\right]\Big|_{r=a}$$

$$= \frac{-q}{4\pi}\left[\frac{-1(2a-2d\cos\theta)}{(a^2+d^2-2ad\cos\theta)^2} + \frac{\frac{q}{d}\left(2a-\frac{2a^2}{d}\cos\theta\right)}{\left(a^2+\left(\frac{a^2}{d}\right)^2-\frac{2a^3}{d}\cos\theta\right)^2}\right]$$

c) $F = \frac{1}{4\pi\epsilon_0}\frac{qq'}{(K-d)^2} = \frac{1}{4\pi\epsilon_0}\left[\frac{-q^2\frac{a}{d}}{\left(\frac{a^2}{d}-d\right)^2}\right] = \frac{1}{4\pi\epsilon_0}\left[\frac{-q^2ad}{(a^2-ad)^2}\right]$

Pulled towards sphere surface. $-\hat{r}$

d) There is no change since the charge is inside the conductor, and information or outside fields is not related in.