13. *Statistical Mechanics and Thermodynamics*

Consider an ideal monoatomic gas in which each atom has two internal energy states, one an energy Δ above the other. There are N atoms in a volume V at temperature T.

Find the a) chemical potential, b) free energy, c) entropy, d) pressure and e) heat capacity at constant pressure.

$$Z_1 = \sum_E e^{-\beta E} = e^{-\beta E} + e^{-\beta(E+\Delta)} = e^{-\beta E} + e^{-\beta E}\, e^{-\Delta}$$

$$= e^{-2\beta E}\, e^{-\Delta}$$

$$Z = \frac{(Z_1)^N}{N!} \qquad \ln Z = N(\ln Z_1 - \ln N + 1)$$

$$= N(-2\beta E - \Delta - \ln N + 1)$$

$$F = -kT \ln Z = -kTN(1 - 2\beta E - \Delta - \ln N)$$

a) $$\mu = \left(\frac{\partial F}{\partial N}\right)_{V,T} = -kT(1 - 2\beta E - \Delta - \ln N) - kTN\left(-\frac{1}{N}\right)$$

$$= kT(2\beta E + \Delta + \ln N - 1) + kT$$

$$= kT(2\beta E + \Delta + \ln N)$$

b) $F = -kT \ln Z$

c) $$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N} = kN(1 - 2\beta E - \Delta - \ln N) + kTN\left(+\frac{2E}{kT^2}\right)$$

$KN(2E\beta)$

$$= kN(1 - \Delta - \ln N)$$

d) $$p = -\left(\frac{\partial F}{\partial V}\right)_{T,N} = 0$$

e) $C_p = Nk\left(1 + \frac{f}{R}\right)$ degrees of freedom $\Rightarrow C_p = Nk\left(1 + \frac{3}{R}\right)$