Fall 2002 #4

4. *Quantum Mechanics.*

Consider the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\nabla^2 - \frac{Ze^2}{r} \tag{1}$$

(a) What is the ground state energy of this Hamiltonian?

(b) What is the expectation value of the potential energy $\left\langle -\frac{Ze^2}{r}\right\rangle$ in the ground state?

(c) What is the expectation value of the kinetic energy $\left\langle -\frac{\hbar^2}{2m}\nabla^2\right\rangle$ in the ground state? ($\hbar = 1$)

Fall 2005 #4

$H = \frac{-\hbar^2}{2m}\nabla^2 - \frac{Ze^2}{r} = \frac{p^2}{2m} - \frac{Ze^2}{r}$

$E = -\frac{Z^2\alpha^2 m}{2n^2}$ $\quad a =$ bohr radius

b) $2\langle T\rangle = \langle r\frac{\partial V}{\partial r}\rangle$ $\quad r\frac{\partial}{\partial r}\left(\frac{-Ze^2}{r}\right) = r\left(\frac{Ze^2}{r^2}\right) = \langle\frac{Ze^2}{r}\rangle$

$\langle\psi|H|\psi\rangle = \langle\psi|T|\psi\rangle + \langle\psi|V|\psi\rangle$

$E_0 = \langle\psi|T|\psi\rangle + \langle\psi|V|\psi\rangle$ $\quad r\frac{\partial V}{\partial r} = -V$

$= \frac{\langle\psi|r\frac{\partial V}{\partial r}|\psi\rangle}{2} + \langle\psi|V|\psi\rangle$

$E_0 = \langle\frac{-V}{2}\rangle + \langle V\rangle$

$2\langle T\rangle = -\langle V\rangle$

$\langle T\rangle = \frac{-\langle V\rangle}{2}$

$E_0 = \frac{1}{2}\langle V\rangle$ $\quad \langle V\rangle = 2E_0 = \frac{-Z^2\alpha^2 m}{n^2}$

c) $\langle T\rangle = -E_0 = \frac{Z^2\alpha^2 m}{2n^2}$