Fall 2002 # 4 (p 1 of 1)

Consider the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\nabla^2 - \frac{Ze^2}{r}$$

(a) what is the ground state energy of this Hamiltonian?

This is just the Hamiltonian for a hydrogen-like atom, Thus, we immediately know the energy levels are (in natural units)

$$\boxed{E_n = -\frac{\alpha^2 m}{2n^2} Z^2} \quad , n = 1, 2, 3 \ldots$$

$n=1$ for ground state

(b) what is the expectation value of the potential energy $\langle -\frac{Ze^2}{r} \rangle$ in the ground state?

from virial theorem (see Griffiths' QM problem 4.41), we know that

$$\langle V \rangle = 2 E_n$$

So,

$$\langle V \rangle \Big|_{n=1} = 2\left(\frac{-\alpha^2 m Z^2}{2n^2}\right)_{n=1} = \boxed{-\alpha^2 m Z^2}$$

(c) what is the expectation value of the kinetic energy $\langle -\frac{\hbar^2}{2m}\nabla^2 \rangle$ in the ground state?

again from virial theorem, we know

$$\langle T \rangle = -E_n$$

So,

$$\boxed{\langle T \rangle = \frac{\alpha^2 m Z^2}{2}}$$