

### 8. Electricity and Magnetism

#### Radiating charges

- (a) A point charge  $q$  under acceleration  $a(t)$  emits electromagnetic radiation. Give qualitative physical arguments why the radiated power,  $P$ , should be of the form  $P = Bq^2a^2$ , where  $B$  is a proportionality constant. Determine by dimensional analysis the dependence of  $B$  on fundamental physical constants. Explain how and why the exact expression for  $B$  differs from this estimate.
- (b) A point charge  $q$  has mass  $m$  and is attached to a spring (of spring constant  $\kappa$ ) hanging from a fixed support above an infinite horizontal conducting plane. The charge is set in motion with amplitude  $A < h$ , the equilibrium height of the charge above the conducting plane. Calculate its instantaneous radiating power.

a) It should have that form, since it matches the radiation of an electric dipole because the monopole does not radiate, but rather its dipole term. It is like it makes its own dipole as it moves.

$$P = \text{Watts} = \frac{J}{s} = \frac{Nm}{s} = B \overset{\text{coulombs}}{C} \frac{m^2}{s^4} = B \frac{A^2 m^2}{s^2}$$

$$B = \frac{Nm s^2}{s^4 m^2} = \frac{Ns}{A^2 m} = \frac{\mu_0}{c} \quad \text{speed of light}$$

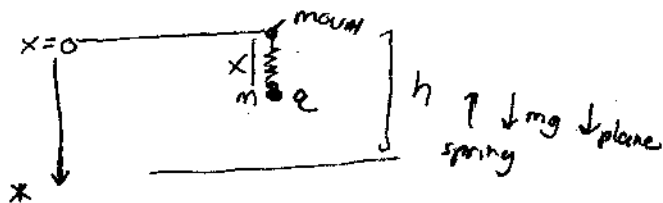
$$P \propto \frac{\mu_0}{c} q^2 a^2 \quad 8$$

The exact expression varies by factors of  $\pi$ 's that can't be found by dimensional analysis

$$b) x(t) = x_0 \cos(\omega t)$$

$$x_0 = A$$

position of  $q$



$$F_s = -kx = -k[A \cos(\omega t)]$$

$$F_{\text{plane}} = -\frac{1}{4\pi\epsilon_0} \frac{q^2}{(2(h-x))^2} \quad \text{Griff. 7hs 3.12}$$

$$F_{\text{gravity}} = mg$$

$$F_{\text{net}} = -k[A \cos(\omega t)] + \frac{1}{4\pi\epsilon_0} \frac{q^2}{(4(h-x)^2)} + mg$$

$$\frac{F_{\text{net}}}{m} = a$$

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} = \frac{\mu_0 q^2}{6\pi c} \left[ \frac{-k[A \cos(\omega t)]}{m} + \frac{1}{4\pi\epsilon_0 m} \frac{q^2}{(4(h-x)^2)} + g \right]^2$$