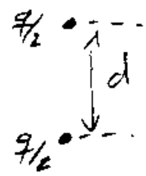
Fall 2002 # 8 (p 1 of 2)

(a) A point charge q under acceleration $a(t)$ emits electromagnetic radiation. Give qualitative physical arguments why the radiated power, P, should be of the form $P = Bq^2a^2$, where B is a proportionality constant. Determine by dimensional analysis the dependence of B on fundamental physical constants. Explain how and why the exact expression for B differs from this estimate.

when a point charge accelerates, there is an imbalance of internal electromagnetic forces (see Griffiths section 11.2.3). You can think of the charge as a dumbell with each the total charge q divided on the two halves separated by a distance d A-like so



the net force on the dumbbell is

$$\vec{F}_{self} = \frac{q}{2}\left(\vec{E}_1 + \vec{E}_2\right) \propto q^2 a^2$$

Now, consider the Larmor Formula (valid when $v \ll c$)

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \Rightarrow \boxed{B = \frac{\mu_0}{6\pi c}}$$

$$\left.\begin{aligned} [P] &= J/s = \frac{Kg\, m^2/s^2}{s} \\ [q^2a^2] &= C^2 (m/s^2)^2 \end{aligned}\right\} \quad \frac{Kg\, m^2}{s^3} = [B]\, \frac{C^2 m^2}{s^4} \Rightarrow \boxed{[B] = \frac{Kg \cdot s}{C^2}} \quad (1)$$

where $C = A \cdot s$

$$[\mu_0] = N/A^2 = \frac{N}{C^2/s^2} = \frac{Kg\, m/s^2}{C^2/s^2} = \frac{Kg \cdot m}{C^2}$$

$$\left[\frac{\mu_0}{c}\right] = \frac{\frac{Kg \cdot m}{C^2}}{m/s} = \frac{Kg \cdot s}{C^2} \quad \checkmark \quad \text{matches eq (1)}$$

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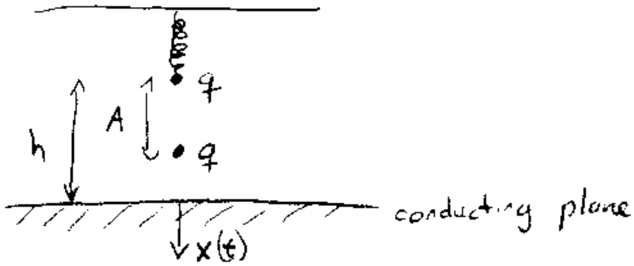
(b) A point charge q has mass m and is attached to a spring (w/ spring constant k) hanging from a fixed support above an infinite horizontal conducting plane. The charge is set in motion with amplitude $A < h$, the equilibrium height of the charge above the conducting plane. Calculate its instantaneous radiating power.



We are told that the charge is set in motion with amplitude A. So set $t=0$ to when charge has amplitude A. That is,

$$x(t) = A\cos(\omega t)$$

where A and ω are a bit complicated due to the force from gravity and image "dipole"

If this was an exact dipole of two charges separated by a distance d the amplitude would be qd.

So, the acceleration is given by

$$a = \ddot{x}(t) = -A\omega^2\cos(\omega t) \qquad (2)$$

Now, from part (a), we are told that the radiated power is

$$P = Bq^2a^2$$

Since we must also consider the image, we have

$$P = 2Bq^2a^2$$

and substituting in for a from eq (2), we have

$$\boxed{P = 2Bq^2A^2\omega^4\cos^2(\omega t)}$$