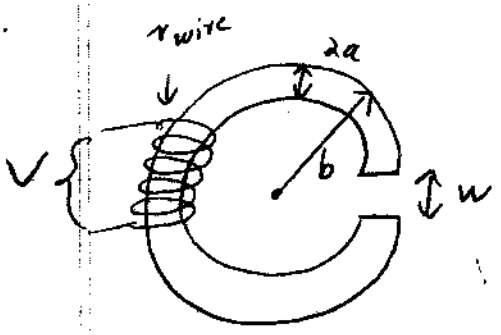
E.M. F'02 #9



$r_{wire} \ll a \ll b$

a) To determine the field in the gap:

$$\oint \vec{H} \cdot d\vec{\ell} = I_{\text{enc}} = NI$$

$$H_{in}(2\pi b - w) + H_{gap}\, w = NI$$

$$H_{in} = \frac{B}{\mu} \quad ; \quad H_{gap} = \frac{B}{\mu_0}$$

so $\frac{B}{\mu}(2\pi b - w) + \frac{B}{\mu_0} w = NI \Rightarrow B = \dfrac{NI}{\frac{1}{\mu}(2\pi b - w) + \frac{w}{\mu_0}}$

$$I = \frac{V}{R} \quad ; \quad R = \rho \frac{N 2\pi a}{\pi r_w^2} = \frac{2Na\rho}{r_w^2} \quad \text{so} \quad B = \frac{V r_w^2}{2a\rho\left[\frac{1}{\mu}(2\pi b - w) + \frac{w}{\mu_0}\right]}$$

b) $\tau = \frac{L}{R} \quad ; \quad L = \frac{N\Phi}{I} \quad ; \quad \Phi = \int B \cdot da = B \cdot \pi a^2$

so $L = \dfrac{NB\pi a^2}{I}$

$$\tau = \frac{L}{R} = \frac{NB\pi a^2}{\underbrace{IR}_{=V}} = \frac{N\pi a^2 B}{V} = \frac{N^2\pi a^2}{\frac{1}{\mu}(2\pi b - w) + \frac{w}{\mu_0}} \cdot \underbrace{\frac{V}{R}}_{\frac{2Na\rho}{r_w^2}} \cdot \frac{1}{V}$$

$$= \frac{N^2\pi a^2}{\frac{1}{\mu}(2\pi b - w) + \frac{w}{\mu_0}} \cdot \frac{r_w^2}{2Na\rho} = \frac{N\pi a r_w^2}{2\rho\left[\frac{1}{\mu}(2\pi b - w) + \frac{w}{\mu_0}\right]}$$

$$= \frac{N\pi a \mu\mu_0 r_w^2}{2\rho\left[\mu_0(2\pi b - w) + \mu w\right]}$$