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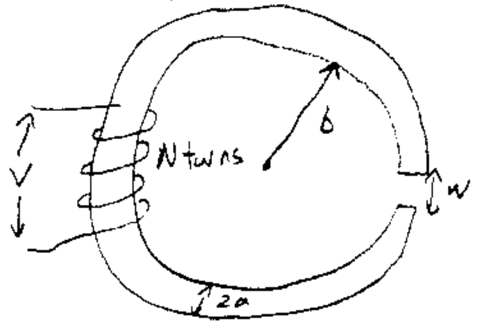
A D.C. electromagnet is constructed from a cylindrical soft-iron bar with radius a. The relative magnetic permeability of the iron is μ. The bar is bent into a C-shape as shown below with radius b. The width of the small gap is w. The magnet is energized by winding a coil of copper wire N turns tightly around the bar and connecting the coil to a D.C. power supply with voltage V. The copper wire has a resistivity ρ and radius r_{wire}. Assume $r_{wire} << a << b$ and ignore fringe-field effects.



(a) what is the steady-state value of the magnetic field B in the gap?

Ampère's circuital law is given by

$$\oint \vec{H} \cdot d\vec{l} = \frac{4\pi}{c} N I$$

$$\Rightarrow \quad H_c (2\pi b - w) + H_G w = N I \left(\frac{4\pi}{c}\right)$$

where H_c is the field from the C-shape magnet and H_G is the field from the gap. So,

$$H_c = \frac{B}{\mu} \quad \text{and} \quad H_G = B$$

Making these substitutions, we get

$$B \left[\frac{1}{\mu}(2\pi b - w) + w \right] = N I \left(\frac{4\pi}{c}\right)$$

$$\Rightarrow \quad B = \frac{\left(\frac{4\pi}{c}\right) N I \mu}{2\pi b - w + w\mu} \qquad (1)$$

Now we want to replace I with what we are given. So, we know that

$$V = IR \Rightarrow I = \frac{V}{R} = \frac{V}{\rho \frac{L}{A}}$$

where L and A are the length and area of the wire. We are told that the radius of the wire is r_w width of the magnet is $2a$ ($\Rightarrow L_w = N 2\pi a$)

$$I = \frac{V \pi r_w^2}{\rho N 2\pi a} = \frac{V r_w^2}{\rho N 2a}$$

Substituting this result into our expression for B yields

$$\boxed{B = \frac{V \mu r_w^2 \left(\frac{4\pi}{c}\right)}{\rho\, 2a \left[2\pi b - w + w\mu\right]}}$$

b) what is the time constant governing the response of the current in the coil when the voltage is turned on?

$$\tau = \frac{L}{R} \quad \text{, where the self inductance , } L \text{, is}$$

$$L = \frac{\Phi_B}{I} = \frac{\int \vec{B} \cdot d\vec{a}}{I}$$

This is actually the flux through a single turn. The flux through N turns is

$$N \int \vec{B} \cdot d\vec{a}$$

So,

$$\tau = \frac{1}{\underbrace{RI}_{=V}} N |\vec{B}| \underbrace{\pi a^2}_{\substack{\text{just where} \\ \text{wires are}}} = \boxed{\frac{N \mu \pi a r_w^2 \left(\frac{4\pi}{c}\right)}{2\rho \left[2\pi b - w + w\mu\right]}}$$