Spring 2002. 1

$a = \frac{1}{\sqrt{2}}(x+ip) \quad a^{\dagger} = \frac{1}{\sqrt{2}}(x-ip)$

$\psi(x) = (2x^3-3x)\, e^{-x^2/2}$

$a^{\dagger}a|\psi_n\rangle = n|\psi_n\rangle \qquad E_n = \hbar\omega(n+1/2)$

$[x,p] = i$

$a^{\dagger}a|\psi\rangle = \frac{1}{2}$

$a^{\dagger}a = \frac{1}{\sqrt{2}}(x-ip)\frac{1}{\sqrt{2}}(x+ip) = \frac{1}{2}(x^2+p^2-1) = \frac{1}{2}\left(x^2 - \frac{\hbar^2\partial^2}{\partial x^2} - 1\right)$ but $\hbar = 1$

$$\frac{\partial}{\partial x}(2x^3-3x)e^{-x^2/2} = -xe^{-x^2/2}(2x^3-3x) + e^{-x^2/2}(6x^2-3)$$

$$\frac{\partial^2}{\partial x^2}(2x^3-3x)e^{-x^2/2} = -e^{-x^2/2}(2x^3-3x) + x^2e^{-x^2/2}(2x^3-3x)$$
$$- xe^{-x^2/2}(6x^2-3) - xe^{-x^2/2}(6x^2-3) + e^{-x^2/2}(12x)$$
$$= x^2e^{-x^2/2}(2x^3-3x) - 2xe^{-x^2/2}(6x^2-3) + e^{-x^2/2}(12x) - e^{-x^2/2}(2x^3-3x)$$

$$\frac{1}{2}e^{-x^2/2}[\cancel{2x^5} - 3x^3 - \cancel{2x^3} + \cancel{3x} - \cancel{2x^5} + 3x^3 + 12x^3 - 6x - 12x + \cancel{2x^3} - \cancel{3x}]$$

$$\frac{1}{2}e^{-x^2/2}[12x^3 - 18x] = [6x^3 - 9x]e^{-x^2/2} = n[2x^3-3x]e^{-x^2/2}$$

$n = 3$

$E_n = \hbar\omega(3+1/2)$

b) Find next two closest states

a^{+} is the raising operator

$$\psi_4 = a^{+}\psi(x) = \frac{1}{\sqrt{2}}(x - ip)\psi(x) = \frac{1}{\sqrt{2}}\left(x - \frac{d}{dx}\right)\psi(x)$$

$$\frac{d}{dx}\psi(x) = e^{-x^2/2}(6x^2-3) - x e^{-x^2/2}(2x^3-3x)$$

$$\psi_4(x) = \frac{1}{\sqrt{2}} e^{-x^2/2}\left[2x^4 - 3x^2 - 6x^2 + 3 + 2x^4 - 3x^2\right]$$

$$= \frac{1}{\sqrt{2}} e^{-x^2/2}\left[4x^4 - 12x^2 + 3\right]$$

$$\psi_2(x) = a\psi(x) = \frac{1}{\sqrt{2}}(x + ip)\psi(x) = \frac{1}{\sqrt{2}}\left(x + \frac{d}{dx}\right)\psi(x)$$

$$= \frac{1}{\sqrt{2}} e^{-x^2/2}\left[\cancel{2x^4} - \cancel{3x^2} + 6x^2 - 3 - \cancel{2x^4} + \cancel{3x^2}\right]$$

$$= \frac{3}{\sqrt{2}} e^{-x^2/2}\left[2x^2 - 1\right]$$