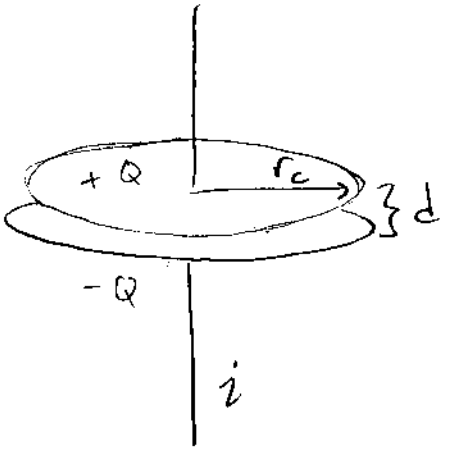
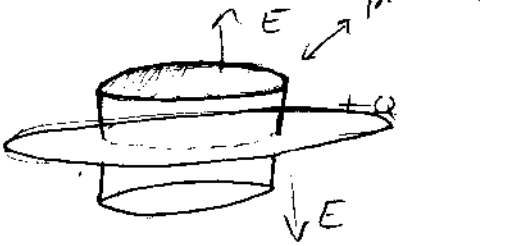
problem #12 Spring 2002



a) put charge $\pm Q$ on plates

$$\sigma_{Top} = \frac{Q}{\pi r_c^2} \qquad \sigma_{bottom} = -\frac{Q}{\pi r_c^2}$$

for one plate



$$\int \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

$$2|E|\pi r^2 = \frac{1}{\varepsilon_0} \frac{Q}{\pi r_c^2} \pi r^2$$

$$E = \frac{\frac{1}{\varepsilon_0} Q}{2\pi r_c^2} = \frac{\sigma}{2\varepsilon_0} \hat{n} \qquad E_{inside} = \frac{\sigma}{\varepsilon_0}$$

$$V = -\int \vec{E} \cdot d\vec{l} = -\frac{\sigma d}{\varepsilon_0} = -\frac{Qd}{A\varepsilon_0}$$

$$Q = CV$$

$$C = \frac{Q}{V} = -\frac{A\varepsilon_0}{d} = \frac{\pi r_c^2 \varepsilon_0}{d}$$

b)

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \int \left(\frac{d\vec{E}}{dt}\right)\cdot d\vec{a}$$

$$\frac{d\vec{E}}{dt} = \frac{d}{dt}\left(\frac{Q}{\pi r_c^2 \varepsilon_0}\right) = \frac{I}{\varepsilon_0 \pi r_c^2}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \cancel{I_{enc}}^{\,0} + \mu_0 \int_0^{r_0} \frac{I}{\pi r_c^2}\cdot d\vec{a}$$

$$\oint \vec{B}\cdot d\vec{l} = |\vec{B}|\, 2\pi r = \mu_0 I$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\phi}}$$

$$\frac{d\vec{E}}{dt} = \frac{I}{\varepsilon_0 A} \Rightarrow \int \left(\frac{d\vec{E}}{dt}\right)\cdot d\vec{a} = \frac{I}{\varepsilon_0}$$

$$\int \frac{I}{\varepsilon_0 A}\cdot d\vec{a} = \frac{I}{\varepsilon_0}$$

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c) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \int \left(\frac{d\vec{E}}{dt}\right)\cdot d\vec{a}$ → 1,0

$$|\vec{B}|\, 2\pi r = \mu_0 I$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\phi}}$$