Spring 2002 #2

$H = A(S_{1z} + S_{2z}) + B\, S_1 \cdot S_2$ Spin 1/2

$S = S_1 + S_2$ $S^2 = S_1^2 + S_2^2 + 2 S_1 \cdot S_2$ $S_1 \cdot S_2 = \frac{(S^2 - S_1^2 - S_2^2)}{2}$

$H = A(S_{1z} + S_{2z}) + B\frac{(S^2 - S_1^2 - S_2^2)}{2}$

$= A(S_{1z} + S_{2z}) + B\left(\frac{S^2}{2} - \frac{3}{4}\right)$

$S^2 |Sm\rangle = S(S+1)$

$S_z |Sm\rangle = 1 |Sm\rangle$

$\frac{1}{2}(\frac{1}{2}+1)$

$\frac{1}{2}(\frac{3}{2}) = \frac{3}{4}$

$S=1$ $\begin{cases} |\uparrow\uparrow\rangle & m=1 \\ \frac{1}{\sqrt{2}}[|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle] & m=0 \\ |\downarrow\downarrow\rangle & m=-1 \end{cases}$

$\frac{1}{2} \otimes \frac{1}{2}$

$S = 1$ or 0

$S=0$ $\frac{1}{\sqrt{2}}[|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle]$ $m=0$

$H = A S_z + B\left(\frac{S^2}{2} - \frac{3}{4}\right)$ since $S_{1z} + S_{2z} = S_z$

$H\psi_{11} = A + \frac{1}{4}B$ $H\psi_{10} = \frac{1}{4}B$ $H\psi_{1-1} = -A + \frac{1}{4}B$

ψ_{sm}

$H\psi_{00} = -\frac{3}{4}B$ Energies $-\frac{3}{4}B,\ \frac{1}{4}B,\ \frac{1}{4}B + A$