Spring 2002 #3 (p 1 of 1)

(a) Two Hermitian operators anticommute. Is it possible for them to have simultaneous eigenstates?

(see Sakurai #1.16)

The only way this is possible is if one of the eigenstates is zero.

That is, for two operators to anticommute.

$$AB + BA = 0$$

consider $A|\psi\rangle = a|\psi\rangle$ and $B|\psi\rangle = b|\psi\rangle$

$$\Rightarrow AB|\psi\rangle = ab|\psi\rangle \quad \& \quad BA|\psi\rangle = ab|\psi\rangle$$

$$\Rightarrow \underbrace{(AB + BA)}_{=0}|\psi\rangle = \underbrace{2ab}_{\text{only equals zero if } a \text{ or } b \text{ is zero}}|\psi\rangle$$

or if $|\psi\rangle$ is zero.

(b) Do position operators at unequal times commute in general in the Heisenberg representation? Give an simple example illustrating your answer.

No. consider a free particle

$$x_i(t) = x_i(0) + v_i(0)t = x_i(0) + \frac{p_i}{m}t$$

$$\Rightarrow [x_i(t), x_i(0)] = \cancel{[x_i(0), x_i(0)]}^{\,0} + \frac{t}{m}[p_i, x_i] = -\frac{i\hbar t}{m} \neq 0$$

(c) Explain how you would interpret the energy-time uncertainty relation.

Δt is the time system takes to change substantially.

Sharper the energy is $\Rightarrow$ larger time scale is

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$