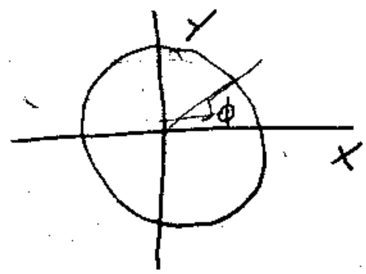
2 Spring 2002 # 5.

$H = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2) + V(r)$

$V(r) = \dot{r} = \dot{\theta} = 0$ since it is stuck on a ring.

$= \frac{1}{2}mr^2\sin^2\theta\,\dot{\phi}^2$ But $\theta = \pi/2$ and $r = R$

$= \frac{1}{2}mr^2\dot{\phi}^2$



$H = \frac{p^2}{2m} = \frac{1}{2}mR^2\dot{\phi}^2 = \frac{L_z^2}{2mR^2}$ $\quad L_z \to -i\frac{\partial}{\partial\phi}$ $\quad L = m\dot{\phi}R$

$\frac{1}{2mR^2}\left(-i\frac{\partial}{\partial\phi}\right)^2 \psi(\phi) = E\psi(\phi)$

$\Rightarrow$ degeneracy 2 per state.

solutions have the form $\psi(\phi) = \frac{1}{\sqrt{2\pi}}e^{in\phi}$, and $\frac{1}{\sqrt{2\pi}}e^{-in\phi}$ is valid

$\frac{1}{2mR^2}\left(\cancel{-}(\cancel{i^2}n^2)\right)\cancel{\frac{e^{in\phi}}{\sqrt{2\pi}}} = E_n\frac{e^{in\phi}}{\sqrt{2\pi}}$ $\quad \boxed{E_n = \frac{n^2}{2mR^2}}$ $\quad n = 1, 2, 3 \ldots$

Can think of $\psi(\phi) = \frac{1}{\sqrt{2\pi}}e^{\pm in\phi}$ as $\psi(\phi) = \frac{1}{\sqrt{2\pi}}e^{in\phi}$ $\quad n = 0, \pm1, \pm2, \pm3$

b) $H' = e\vec{E}\cdot\vec{r} = eE_0R\cos\phi = eE_0x$

$\langle\psi_n^0|eE_0x|\psi_n^0\rangle = 0$ since x is odd parity

and $\Rightarrow$ The bra and kets can't be the same parity for odd operators $\Rightarrow 0$

c) 2^{nd} order shift

$$\Delta E^{(2)} = \sum_{m\neq n} \frac{|\langle \psi_m^0 | H' | \psi_n^0 \rangle|^2}{E_n^0 - E_m^0}$$

$$\langle \psi_m^0 | H' | \psi_n^0 \rangle = \frac{1}{2\pi}\int_0^{2\pi} e^{-im\phi}\, e E_0 r \cos\phi\, e^{in\phi}\, d\phi$$

$$\cos\phi = \frac{e^{i\phi} + e^{-i\phi}}{2}$$

$$\langle \psi_m^0 | H' | \psi_n^0 \rangle = \frac{eE_0 r}{4\pi}\int_0^{2\pi} e^{i\phi(-m+n+1)} + e^{i\phi(-m+n-1)}\, d\phi$$

$$= \frac{eE_0 r}{4\pi}(2\pi) \quad \text{for } m = n\pm 1, \quad 0 \text{ otherwise}$$

$$= \frac{eE_0 r}{2}$$

$$\Delta E^{(2)} = \frac{|\langle \psi_{n+1} | H' | \psi_n \rangle|^2}{E_n^0 - E_{n+1}^0} + \frac{|\langle \psi_{n-1} | H' | \psi_n \rangle|^2}{E_n^0 - E_{n-1}^0}$$

$E = \frac{n^2}{2mr^2}$

$$= \frac{e^2 E_0^2 r^2}{4\frac{1}{(2mr^2)}[n^2 - (n+1)^2]} + \frac{e^2 E_0^2 r^2}{4\frac{1}{(2mr^2)}[n^2 - (n-1)^2]}$$

$$= \frac{e^2 E_0^2 m r^4}{2\,[n^2 - n^2 - 2n - 1]} + \frac{e^2 E_0^2 m r^4}{2\,[n^2 - n^2 + 2n - 1]}$$

$$= \frac{e^2 E_0^2 m r^4}{2}\left[\frac{1}{(-2n-1)} + \frac{1}{(2n-1)}\right] = \frac{e^2 E_0^2 m r^4}{2}\left[\frac{(2n-1)}{(-2n-1)(2n-1)} + \frac{(-2n-1)}{(-2n-1)(2n-1)}\right]$$

$$= \frac{e^2 E_0^2 m r^4}{2}\left[\frac{2}{4n^2-1}\right] = \frac{e^2 E_0^2 m r^4}{(4n^2-1)}$$