Spring 2002 #6 (p 1 of 1)

Calculate the collision frequency for the collisions between the molecules of a gas and a fixed sphere of diameter D. The molecules have an average diameter d. The gas has a temperature T.

(See spring 2001 #6)

maxwell distribution function: $D(v) = \underbrace{\left(\frac{m}{2\pi kT}\right)^{3/2}}_{\text{normalization condition}} 4\pi r^2 \underbrace{e^{-\frac{\beta m v^2}{2}}}_{\text{probability of a molecule having } \vec{v}}$

mean molecular speed

$$v_{average} = \sum_v v\, D(v)\, dv \Rightarrow \int_0^\infty \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^3 e^{-mv^2/2kT}\, dv$$

$$\text{let } x = v^2 \Rightarrow dx = 2v\, dv$$

So, we have

$$v_{average} = \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty 2\pi\, x\, e^{-mx/2kT}\, dv = 2\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty x e^{\frac{-mx}{2kT}}\, dv$$

$$= 2\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{1!}{(m/2kT)^2} = 2\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \left(\frac{2kT}{m}\right)^2$$

$$v_{average} = \frac{2\pi}{\pi^{3/2}} \left(\frac{2kT}{m}\right)^{1/2} = \sqrt{\frac{8kT}{m\pi}} \qquad (1)$$

← equation 12.3.10 in Reif

Now, we have

$$\nu = \frac{v_{average}}{l} \approx \frac{\sqrt{\frac{8kT}{m\pi}}}{\frac{4}{\pi(d+D)^2 n}} = \sqrt{\frac{8kT}{16 m\pi}}\, \pi (d+D)^2 n$$

$$\therefore \boxed{\nu = n(d+D)^2 \sqrt{\frac{kT\pi}{2m}}}$$