

Problem # 11 Fall 2003

$$k^2 c^2 = \omega^2 - 4\pi n e^2 / m_e$$

$$c = 3 \times 10^{10} \quad m_e = 9.11 \times 10^{-29} \quad e = 4.8 \times 10^{-10}$$

a) For Transmission

$$\omega^2 - \frac{4\pi n e^2}{m_e} \geq 0$$

otherwise k is imaginary and the radiation is absorbed.

$$\Rightarrow \frac{4\pi n e^2}{m_e} = \omega^2 \quad \text{is the frequency}$$

where transmission stops.

$$n = \frac{\omega^2 m_e}{4\pi e^2} = \frac{(2\pi \cdot 10^7)^2 (9.11 \times 10^{-29})}{4\pi (4.8 \times 10^{-10})^2}$$

$$n \approx \frac{\pi}{25} \frac{10^{14} \times 10^{-27}}{10^{-20}} = \frac{\pi}{25} 10^7$$

So from the information given, one might

conclude that Radiation with frequencies below 10 MHz can't be received from space because the earth is surrounded by a plasma of density $n \approx \frac{\pi}{25} 10^7$

I looked on the internet, and indeed this is the case.

$$b) \quad k = \frac{\omega}{v} \quad n = 0.01 \text{ cm}^{-3} \quad d = 1 \times 10^{22} \text{ cm}$$

$$d = v t \quad t = \frac{d}{v} \quad \omega_2 = 10 \text{ kHz} \quad \omega_1 = 6 \text{ kHz}$$

$$\frac{1}{v} = \frac{1}{c} \sqrt{1 - \frac{4\pi n e^2}{\omega^2 m_e}}$$

$$t = \frac{d}{v} = \frac{d}{c} \sqrt{1 - \frac{4\pi n e^2}{\omega^2 m_e}}$$

$$t_2 - t_1 = \frac{d}{c} \left\{ \sqrt{1 - \frac{4\pi n e^2}{\omega_2^2 m_e}} - \sqrt{1 - \frac{4\pi n e^2}{\omega_1^2 m_e}} \right\}$$