A free particle of mass m, travelling with momentum p parallel to the z-axis, scatters off the potential

$$V = V_0 \left[\delta(\vec{r} - a\hat{z}) - \delta(\vec{r} + a\hat{z}) \right].$$

Compute the differential cross section, $\frac{d\sigma}{d\Omega}$ in the Born approximation.

(see Zettili problem 11.2, p 618)

First re-write the delta functions as

$$\delta(\vec{r} \pm a\vec{z}) = \delta(x)\,\delta(y)\,\delta(z \pm a)$$

Now, we recognize that this is not a spherically symmetric potential. So the first Born approximation scattering amplitude is then (Abers eq 8.36)

$$f^{(1)}(\theta,\phi) = -\frac{2m}{4\pi} \int d^3r \; V(r)\, e^{i\vec{q}\cdot\vec{r}} \qquad (1)$$

dr

where $\vec{q}$ is the momentum transfer defined as

$$\vec{q} = \vec{k} - \vec{k}'$$

then

$$q^2 = |\vec{k}|^2 + |\vec{k}'|^2 - 2\vec{k}\cdot\vec{k}$$

Since this is an elastic collision, $|\vec{k}'| = |\vec{k}|$. So,

$$q^2 = 2k^2(1 - \cos\theta) = 4k^2 \sin^2\left(\frac{\theta}{2}\right) \qquad (2)$$

Substituting our expression for $V(r)$ into eq (1) yields

$$f^{(1)}(\theta,\phi) = \frac{-2m}{4\pi} V_0 \int dx\, \delta(x)\, e^{iq_x x} \int dy\, \delta(y)\, e^{iq_y y} \int dz \left(\delta(z-a) - \delta(z+a) \right) e^{iq_z z}$$

$$= -\frac{mV_0}{2\pi}\left[e^{iq_z a} - e^{-iq_z a} \right] = \frac{-mV_0}{\pi}\, i \,\sin(q_z a)$$

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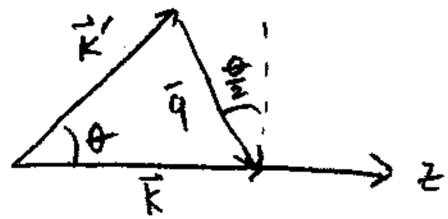
but, what is q_z ?



$\leftarrow q_z = q \sin\left(\frac{\theta}{2}\right)$

From eq (2), we have an expression for q. So,

$$q_z = 2K \sin^2\left(\frac{\theta}{2}\right)$$

Substituting this result into our expression for the scattering amplitude yields

$$f^{(1)}(\theta,\phi) = \frac{-mV_0}{\pi} i \sin\left[2Ka\sin^2\left(\frac{\theta}{2}\right)\right]$$

Then, the differential cross section is

$$\frac{d\sigma}{d\Omega} = \left|f^{(1)}(\theta,\phi)\right|^2$$

$$\therefore \boxed{\frac{d\sigma}{d\Omega} = \frac{m^2 V_0^2}{\pi^2} \sin^2\left[2Ka\sin^2\left(\frac{\theta}{2}\right)\right]}$$