Problem # 3 Fall 2003

$$V(x) = \begin{cases} \frac{1}{2} m\omega^2 x^2, & x > 0 \\ \infty, & \text{otherwise} \end{cases}$$

a) $E_n = \hbar\omega\left((2n+1) + \frac{1}{2}\right) = \hbar\omega\left(2n + \frac{3}{2}\right)$

$$\boxed{E_0 = \hbar\omega\left(\frac{3}{2}\right) = \frac{3}{2}\hbar\omega}$$

b) $x = \left(\frac{\hbar}{2m\omega}\right)^{1/2}\left(\hat{a} + \hat{a}^\dagger\right)$

$$x^2 = \frac{\hbar}{2m\omega}\left(aa + aa^\dagger + \underbrace{a^\dagger a}_{N} + a^\dagger a^\dagger\right)$$

$$[a, a^\dagger] = aa^\dagger - a^\dagger a = 1$$

$$aa^\dagger = N + 1$$

$$x^2 = \frac{\hbar}{2m\omega}\left(aa + 2N + 1 + a^\dagger a^\dagger\right)$$

$$\langle x^2 \rangle = \frac{\hbar}{2m\omega}\langle 2n+1|(2N+1)|2n+1\rangle = \frac{\hbar}{2m\omega}\left[2(2n+1)+1\right]$$

$$\boxed{\langle x^2 \rangle = \frac{\hbar}{2m\omega}(4n+3) = \frac{\hbar}{m\omega}\left(2n + \frac{3}{2}\right)}$$

$\langle T \rangle = \frac{3}{4}\hbar\omega = \frac{1}{2}E$

$\frac{3}{2}\hbar/m\omega = \langle T \rangle$

$\langle x^2 \rangle = \frac{2}{m\omega^2}\langle T \rangle$