Fall 2003 #3 (p 1 of 2)

Consider a particle moving in the potential

$$V(x) = \begin{cases} \frac{1}{2} m\omega^2 x^2 & x > 0 \\ \infty & \text{otherwise} \end{cases}$$

(a) what is the lowest energy eigenvalue?

(See Zettili problem 4.9, p253)

This is an unsymmetric harmonic oscillator potential. So, we must have the wave function vanish at $x=0$. So, those solutions must be those of an ordinary (symmetric) harmonic oscillator that have odd parity since only odd solutions vanish at the origin.

So, since we already know the energies of a symmetric harmonic oscillator

$$E_n = (n + \tfrac{1}{2})\,\omega$$

Then the energies of this unsymmetric potential must be given by those corresponding to the odd n energy levels of ~~the~~ symmetric potential. That is,

$$E_n = \left[(2n+1) + \tfrac{1}{2}\right]\omega$$

$$\therefore \quad E_n = \left[2n + \tfrac{3}{2}\right]\omega$$

So, the lowest energy eigen value is

$$\boxed{E_0 = \frac{3}{2}\,\omega}$$

(b) what is $\langle x^2 \rangle$?

From the virial theorem for harmonic oscillators, we know that

$$\langle V \rangle = \frac{E_n}{2}$$

Since $\langle V(x)\rangle = \frac{1}{2}m\omega^2\langle x^2\rangle$, Then $\langle x^2\rangle = \frac{2}{m\omega^2}\langle V\rangle$

and thus,

$$\langle x^2\rangle = \frac{2}{m\omega^2}\frac{E_n}{2} = \frac{E_n}{m\omega^2}$$

$$\Rightarrow \boxed{\langle x^2\rangle = \frac{\left(2n+\frac{3}{2}\right)}{m\omega}}$$

For lowest energy, we have

$$\langle x^2\rangle = \frac{3}{2m\omega}$$