Fall 2003 #5 (p 1 of 2)

A sample of hydrogen atoms in the ground state is placed between the plates of a parallel capacitor. A voltage pulse is applied to the capacitor at $t=0$ to produce a homogeneous electric field, $\mathcal{E}$, between the plates of:

$$\mathcal{E}=0 \qquad t<0$$
$$\mathcal{E}=\mathcal{E}_0 e^{-t/\tau} \qquad t>0$$

where τ is a constant. A long time compared to τ passes $(t \gg \tau)$

(a) To first order, calculate the fraction of atoms in the $2p$ $(m=0)$ state

once again, this is a time dependent perturbation problem. the general form of the transition probability is given by Zettili eq 10.41 (see also Spring 2003 #1)

$$P_{if}(t) = \left| -i \int_0^t \langle \psi_f | V'(t') | \psi_i \rangle e^{i\omega_{fi} t'} dt' \right|^2 \qquad (1)$$

where $V'(t')$ is given by

$$V'(t') = e\,\mathcal{E}_0 e^{-t'/\tau}\, z$$ ← time dependent stark effect

So, since $t \gg \tau$

$$P_{if}(t) = e^2 \mathcal{E}_0^2 \left| \int_0^\infty \langle \psi_f | z | \psi_i \rangle e^{(i\omega_{fi} - \frac{1}{\tau})t'} dt' \right|^2$$

where $\omega_{fi} = E_f - E_i = -\frac{\alpha^2 m}{2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

now, before we start, note that selection rules for

$$\langle n'\ell' m' | z | n \ell m \rangle \neq 0 \qquad (2)$$

tell us that $|\Delta \ell| = 1$ and $|\Delta m| = 0$ since z is odd and a rank one tensor.

So, for part a we want to find the 2p (m=0) state. So, we want

$$\underbrace{\langle 210|z|100\rangle}_{\text{selection rules tell us this will not vanish}} = \int R_{21}^{*} Y_1^{0*}\, z\, R_{10} Y_0^{0}\, d^3r \quad , \; z = r\cos\theta$$

$$= \int \left[\frac{1}{2\sqrt{6}} \frac{r}{a^{5/2}} e^{-r/2a}\right]\left[\sqrt{\frac{3}{4\pi}}\cos\theta\right](r\cos\theta)\left[\frac{2}{a^{3/2}} e^{-r/a}\right]\left[\frac{1}{\sqrt{4\pi}}\right] d^3r$$

$$= \frac{a^{-4}}{4\pi\sqrt{2}} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \cos^2\theta \int_0^{\infty} r^4 e^{-3r/2a}\, dr$$

let $u = \cos\theta \Rightarrow du = -\sin\theta\, d\theta$

$$= \frac{a^{-4}}{4\pi\sqrt{2}} 2\pi \left(\int_{-1}^{1} du\, u^2\right)\left(\frac{4!}{\left(\frac{3}{2a}\right)^5}\right) = \frac{a\, 2^5 \cdot 2^3 \cdot 3}{2\sqrt{2}\, 3^5}\left[\frac{u^3}{3}\right]_{-1}^{1}$$

$$= \frac{a\, 2^8}{\sqrt{2}\, 3^5}$$

So, then the transition probability is

$$P = e^2 \mathcal{E}_0^2 \frac{a^2 2^{15}}{3^{10}} \left| \int_0^{\infty} e^{\left(i\omega_{fi} - \frac{1}{\tau}\right)t'} dt' \right|^2$$

Thus,

$$\boxed{P = \frac{2^{15}}{3^{10}} \frac{e^2 \mathcal{E}_0^2 a^2}{\omega^2 + \left(\frac{1}{\tau}\right)^2}} \qquad \text{for } \omega = \frac{-\alpha^2 m}{2}\left(\frac{1}{4} - 1\right) = \frac{3\alpha^2 m}{8}$$

(b) to first order, what is the fraction of atoms in the 2s state ($|200\rangle$)

selection rules tell that $\langle 200|z|100\rangle = 0$ since $|\Delta l| \neq 1$.

Thus,

$$\boxed{P_{1s\to 2s} = 0}$$