QM S'03 #3

A coherent state of a simple harmonic oscillator is an eigenstate of the annihilation operator, a. In terms of the energy eigenvalues basis, give an explicit expression for a coherent state $|\alpha\rangle$ satisfying $a|\alpha\rangle = \alpha|\alpha\rangle$.

We have

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

expand $|\alpha\rangle$ in the energy basis:

$$|\alpha\rangle = \sum c_n |n\rangle$$

now $\quad c_n = \frac{\alpha}{\sqrt{n}} c_{n-1} = \frac{\alpha^n}{\sqrt{n!}} c_0$

to properly normalize the wavefunction

$$1 = \langle\alpha|\alpha\rangle = |c_0|^2 \sum \frac{|\alpha|^{2n}}{n!} = |c_0|^2 e^{|\alpha|^2} \quad \text{as } e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots = \sum_n \frac{x^n}{n!}$$

so $\quad c_0 = e^{-\frac{|\alpha|^2}{2}}$

Hence

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$