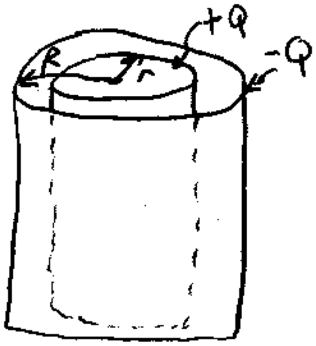
A cylindrical capacitor of length L is composed of an inner cylindrical conductor of radius r and a concentric outer conducting cylindrical shell of radius R.

(a) what is the capacitance of the arrangement?



the capacitance is given by $C = \frac{Q}{V}$. (1)

where

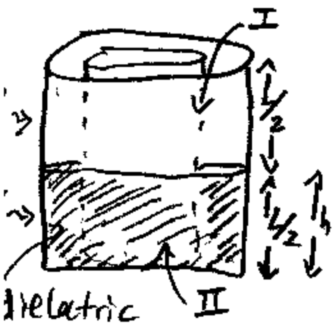
$$V = -\int_R^r \vec{E}\cdot d\vec{l} \quad , \quad \oint \vec{E}\cdot d\vec{a} = 4\pi Q \Rightarrow |\vec{E}| = \frac{4\pi Q}{2\pi s L} = \frac{2Q}{sL}$$

$$\Rightarrow V = -\int_R^r \frac{2Q}{sL}\,ds = -\frac{2Q}{L}\ln\frac{r}{R} = \frac{2Q}{L}\ln\frac{R}{r} \qquad (2)$$

Thus,

$$C = \frac{Q}{V} = \frac{Q}{\frac{2Q}{L}\ln(\frac{R}{r})} = \boxed{\frac{L}{2\ln(R/r)}}$$

(b) The two conductors are held at constant potential difference, V, using a battery. A cylindrical shell of dielectric length L and which just fits between the conductors is inserted so that half is inside the conductor. What is the force on the dielectric in this position (see Fall 1997 #2)



the will be a force on the dielectric since the capacitance changes of the form

$$F = \frac{V^2}{2}\frac{dC}{dh} \qquad \text{(see Griffiths eq 4.64)}$$

since $C = \frac{Q}{V}$, we need to find Q and V for this arrangement.

(i) $Q = \lambda(L-h) + \lambda' h$ (2) ← need to keep general to allow dielectric to move!

(ii) From eq (2), we have that (with $Q \to \lambda\frac{L}{2}$ and $L \to L/2$)

$$V_I = 2\lambda \ln\frac{R}{r}$$

and

$$V_{II} = \frac{2\lambda'}{\epsilon}\ln\frac{R}{r} \qquad \leftarrow \vec{E} = \frac{\vec{D}}{\epsilon}$$

Since $V_I = V_{II}$, we have

$$2\lambda \ln \frac{R}{r} = \frac{2\lambda' \ln \frac{R}{r}}{\epsilon} \quad \Rightarrow \quad \lambda' = \epsilon \lambda \qquad (4)$$

inserting this result into eq (3) yields

$$Q = \lambda(L - h + \epsilon h) = \lambda\left[L + h(\epsilon - 1)\right]$$

note : $\epsilon - 1 = 4\pi \chi_e$

so,

$$Q = \lambda\left(L + h 4\pi \chi_e\right)$$

Thus,

$$C = \frac{Q}{V_I} = \frac{\lambda(L + h4\pi\chi_e)}{2\lambda \ln(R/r)} = \frac{L + h4\pi\chi_e}{2\ln(R/r)}$$

and

$$F = \frac{V^2}{2}\frac{dC}{dh} = \frac{V^2}{2}\frac{4\pi\chi_e}{2\ln(R/r)}$$

$$\Rightarrow \boxed{F = \frac{V^2 \pi \chi_e}{\ln(R/r)}}$$

dielectric will rise until $\frac{dC}{dh} = 0$