Fall 2004 #1

$$\langle\psi|(\vec{S}_a\cdot\hat{n}_a)(\vec{S}_b\cdot\hat{n}_b)|\psi\rangle$$

$$= \sum_{ij}(n_a)_i(n_b)_j\langle\psi|(S_a)_i(S_b)_j|\psi\rangle$$

$$\langle\psi|\tfrac{1}{3}\delta_{ij}\sum_k(S_a)_k(S_b)_k|\psi\rangle + \ldots$$

abers eq. 5.40

(This is expanding $(S_a)_i(S_b)_j$ into traceless matrices)

Only need first term since $(S_a)_i(S_b)_j$ is a scalar

$$\rightarrow \langle\psi|(S_a\cdot\hat{n}_a)(S_b\cdot\hat{n}_b)|\psi\rangle = \tfrac{1}{3}\hat{n}_a\cdot\hat{n}_b\langle\psi|S_a\cdot S_b|\psi\rangle$$

But $S^2=0$ $\quad S=S_a+S_b$

$$S_a\cdot S_b=\tfrac{1}{2}(S^2-S_a^2-S_b^2)=-\tfrac{3}{4}$$

$$\langle\psi|(S_a\cdot\hat{n}_a)(S_b\cdot\hat{n}_b)|\psi\rangle=-\tfrac{1}{3}\cos\theta\,\tfrac{3}{4}=-\tfrac{1}{4}\cos\theta$$