Spring 2004 #11 (p 1 of 1)

Consider a plasma of free charges of mass m and charge e at constant density n. What is the index of refraction for electromagnetic waves of frequency ω which are incident upon this plasma? (see Spring 2003 #10)

the index of refraction of a plasma is given by

$$n = \sqrt{1 + \chi_e} \qquad (1)$$

where χ_e can be found from the induced polarization. where

$$P = \chi_e E = np \qquad (2)$$

, p is the dipole moment and n is the density

where p is

$$p = ex \qquad (3)$$

So, what is x? x can be found from the equation of motion. That is, we have

$$m\ddot{x} = eE_0 e^{-i\omega t} = eE \qquad (4)$$

$$\Rightarrow x = x_0 e^{-i\omega t} \qquad \Rightarrow \ddot{x} = -\omega^2 x$$

So, substituting this result back into eq (4) yields

$$m\omega^2 x = -eE \quad \Rightarrow \quad x = \frac{-eE}{m\omega^2}$$

substituting this result into eq 3, then p into eq (2) yields

$$P = \chi_e E = -\frac{ne^2 E}{m\omega^2}$$

thus,

$$\chi_e = \frac{-ne^2}{m\omega^2} = -\frac{\omega_p^2}{\omega^2} \quad , \text{where } \omega_p^2 = \frac{ne^2}{m}$$

Finally

$$\boxed{n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}}$$