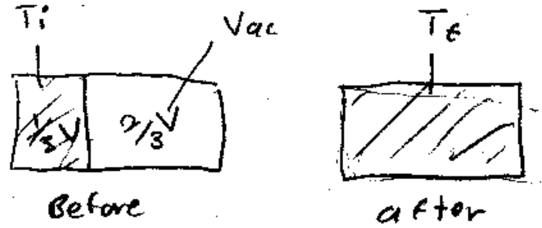
Spring 2004. #13

$$p(T,V) = \frac{NkT}{(V-bN)} - a\left(\frac{N}{V}\right)^2$$



volume $\frac{V}{N}$

$C_V = \frac{3}{2}Nk$ since the same as an ideal gas.

$$dE = TdS - pdV = d(TS) - SdT - pdV$$

work do by system

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{Nk}{(V-bN)} \qquad \left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

$$= \frac{TNk}{(V-bN)} - p = a\left(\frac{N}{V}\right)^2$$

$C_V(T) \rightarrow$ since it is the same as an ideal gas

$$dE = C_V\,dT + a\left(\frac{N}{V}\right)^2 dV \qquad \text{eq 5.8.10 Reif}$$

Since

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV \qquad \left(\frac{\partial S}{\partial T}\right)_V = \frac{1}{T}C_V$$

$$dS = \frac{C_V}{T}dT + \left(\frac{\partial p}{\partial T}\right)_V dV \qquad \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

plug into

$$dE = TdS - pdV$$

Side note

$$E(T,V) = \int_{T_0}^{T} C_V(T')\,dT' - a\left(\frac{N^2}{V}\right) + \text{constant}$$

$$= C_V T - \frac{aN^2}{V} + \text{constant}$$

In a free expansion $\Rightarrow \Delta Q = 0$

$\Delta W = 0$

$\boxed{\Delta E = 0}$

$$E(T_2, V_2) = E(T_1, V_1)$$

$$\int_{T_0}^{T_2} C_V(T')\,dT' - \frac{aN^2}{V_2} = \int_{T_0}^{T_1} C_V(T')\,dT' - \frac{aN^2}{V_1}$$

$$\Rightarrow \int_{T_0}^{T_2} C_V(T')\,dT' - \int_{T_0}^{T_1} C_V(T')\,dT' = a\left(\frac{N^2}{V_2} - \frac{N^2}{V_1}\right)$$

$$\int_{T_1}^{T_2} C_V(T')\,dT' = aN^2\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

Van der Wall gas $T_1 < T' < T_2$ has a constant specific heat at fixed volume

$$\Rightarrow C_V(T_2 - T_1) = aN^2\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

$$T_2 - T_1 = -\frac{aN^2}{C_V}\left(\frac{1}{V_1} - \frac{1}{V_2}\right) \qquad C_V = \frac{3}{2}NK$$

$$T_2 = T_1 - \frac{2aN^2}{3NK}\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$$

$$\boxed{T_f = T_i - \frac{2aN}{3K}\left(\frac{1}{V_1} - \frac{1}{V_2}\right)}$$

when $a=0$ we get the free expansion of an ideal gas.

$$= T_i - \frac{2aN}{3K}\left(\frac{3}{V} - \frac{1}{V}\right)$$

$$T_f = T_i - \frac{4aN}{3KV} \qquad V_1 = \tfrac{1}{3}V, \quad V_2 = V$$