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The table below shows some Clebsch-Gordan coefficients. If two particles have spin 1/2 and 3/2 respectively, write down all composite states $|s\,m\rangle$ in terms of the uncoupled states using Dirac notation.

The possible values of s is given by

$$|s_1 - s_2| \le s \le |s_1 + s_3|$$

$$\Rightarrow \left|\tfrac{1}{2} - \tfrac{3}{2}\right| \le s \le \left|\tfrac{1}{2} + \tfrac{3}{2}\right| \quad \Rightarrow \quad 1 \le s \le 2$$

Thus, s can be either 1 or 2.

if you want to read a column on the table, it has the form

$$|s\,m\rangle = \sum_{m_1 + m_2 = m} C^{s_1 s_2 s}_{m_1 m_2 m} \, |s_1 m_1\rangle |s_2 m_2\rangle$$

where

$$\begin{array}{c|c} s_1 \times s_2 & \begin{array}{c} s \\ m \end{array} \\ \hline m_1\, m_2 & \left(C^{s_1 s_2 s}_{m_1 m_2 m}\right)^2 \end{array}$$

So, for $s=2$ $\left(s_1 = \tfrac{3}{2},\ s_2 = \tfrac{1}{2}\right)$

$$|2\,2\rangle = |\tfrac{3}{2}, \tfrac{3}{2}\rangle\,|\tfrac{1}{2}, \tfrac{1}{2}\rangle$$

$$|2\,1\rangle = \frac{1}{\sqrt{4}}\,|\tfrac{3}{2}, \tfrac{3}{2}\rangle\,|\tfrac{1}{2}, -\tfrac{1}{2}\rangle + \sqrt{\tfrac{3}{4}}\,|\tfrac{3}{2}, \tfrac{1}{2}\rangle\,|\tfrac{1}{2}, \tfrac{1}{2}\rangle$$

$$|2\,0\rangle = \frac{1}{\sqrt{2}}\,|\tfrac{3}{2}, \tfrac{1}{2}\rangle\,|\tfrac{1}{2}, -\tfrac{1}{2}\rangle + \frac{1}{\sqrt{2}}\,|\tfrac{3}{2}, -\tfrac{1}{2}\rangle\,|\tfrac{1}{2}, \tfrac{1}{2}\rangle$$

$$|2, -1\rangle = \sqrt{\tfrac{3}{4}}\,|\tfrac{3}{2}, -\tfrac{1}{2}\rangle\,|\tfrac{1}{2}, -\tfrac{1}{2}\rangle + \frac{1}{\sqrt{4}}\,|\tfrac{3}{2}, -\tfrac{3}{2}\rangle\,|\tfrac{1}{2}, \tfrac{1}{2}\rangle$$

$$|2, -2\rangle = |\tfrac{3}{2}, -\tfrac{3}{2}\rangle\,|\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

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) For $s=1$

$$|1\,1\rangle = \sqrt{\tfrac{3}{4}}\,|3/2, 3/2\rangle\,|\tfrac{1}{2}, -1/2\rangle - \frac{1}{\sqrt{4}}\,|3/2, 1/2\rangle\,|1/2, 1/2\rangle$$

$$|1\,0\rangle = \frac{1}{\sqrt{2}}\,|3/2, 1/2\rangle\,|1/2, -1/2\rangle - \frac{1}{\sqrt{2}}\,|3/2, -1/2\rangle\,|3/2, \tfrac{1}{2}\rangle$$

$$|1, -1\rangle = \frac{1}{\sqrt{4}}\,|3/2, -1/2\rangle\,|\tfrac{1}{2}, -1/2\rangle - \sqrt{\tfrac{3}{4}}\,|3/2, -3/2\rangle\,|1/2, 1/2\rangle$$