A hydrogen atom is in the ground state ($n=1, \ell=m=0$) for $t<0$. Suppose the atom is placed between the plates of a capacitor, and a weak, spatially uniform but time-dependent decaying field is applied at $t=0$. The field (for $t>0$) is

$$\vec{E} = \vec{E}_0 e^{-\gamma t}$$

for some $\gamma > 0$. Take $E_0$ along the z-axis. What is the probability (to first order in $E_0$) that the atom will be in each of the four $n=2$ states as $t \to \infty$? Neglect spin.

this is a time dependent perturbation problem. (see also Fall 2003 #5 and Spring 2003 #1)

the transition probability for $t \to \infty$ is given by Zettili eq 10.11

$$P_{fi}(t) = \left| \int_0^\infty \langle \psi_f | V'(t') | \psi_i \rangle e^{-i\omega_{fi} t'} dt' \right|^2$$

where

$$V'(t') = e E_0 e^{-\gamma t} z$$ ⇐ time dependent stark effect

and

$$\omega_{fi} = E_f - E_i = -\frac{\alpha^2 m}{2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\Bigg|_{\substack{n_f=2\\ n_i=1}} = \frac{3\alpha^2 m}{8}$$

For

$$\langle \psi_f | V'(t') | \psi_i \rangle = e E_0 e^{-\gamma t} \langle 2 \ell' m' | z | 100 \rangle$$

we know the following selection rules since $z$ is odd and a first rank tensor. for the matrix element to be non zero, we need $|\Delta \ell| = 1$ and $|\Delta m| = 0$

Thus, $\ell' = 1$ and $m' = 0$. The other elements vanish.

From Fall 2003 #5, we know that

$$\langle 210 | z | 100 \rangle = \frac{a\, 2^8}{\sqrt{2}\, 3^5}$$

so,

$$P(t) = \frac{e^2 E_0^2 a^2 2^{15}}{3^{10}} \left| \int_0^\infty e^{-(i\omega_{21}+\gamma)t'} dt' \right|^2$$

$$\Rightarrow \boxed{P(t) = \frac{e^2 a^2 E_0^2 \, 2^{15}}{3^{10}(\omega_{21}^2+\gamma^2)}} \quad , \omega_{21} = \frac{3\alpha^2 m}{8}$$