The normalized wave function of a one-dimensional particle is

$$\psi(x) = N e^{-Kx^2/2}$$

for some $K>0$, N is real and positive.

(a) what is N?

use normalization condition to find N.

$$1 = |N|^2 \int_{-\infty}^{\infty} e^{-Kx^2}\, dx = |N|^2 \sqrt{\frac{\pi}{K}} \Rightarrow \boxed{N = \left(\frac{K}{\pi}\right)^{1/4}}$$

(b) what is expectation value of x^2?

$$\langle x^2 \rangle = |N|^2 \int_{-\infty}^{\infty} x^2 e^{-Kx^2}\, dx = |N|^2 \frac{\sqrt{\pi}}{2K^{3/2}} = \sqrt{\frac{K}{\pi}} \frac{\sqrt{\pi}}{2K^{3/2}} = \boxed{\frac{1}{2K}}$$

(c) what is the momentum space wave function $\langle p|\psi\rangle$?

(see Abers eq 2.192) ($\hbar = 1$)

$$\langle p|\psi\rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ipx}\, \psi(x)\, dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ipx} N e^{-Kx^2/2}\, dx$$

$$= \frac{N}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left(\frac{K}{2}x^2 + ipx\right)}\, dx$$

note: $\displaystyle\int_{-\infty}^{\infty} e^{-(at^2 - bt + c)}\, dt = \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2 - 4ac}{4a}\right)$

So,

$$\langle p|\psi\rangle = \frac{N}{\sqrt{2\pi}} \sqrt{\frac{2\pi}{K}}\, e^{-p^2/2K} = \left(\frac{K}{\pi}\right)^{1/4} \frac{1}{K^{1/2}}\, e^{-p^2/2K}$$

$$\therefore \boxed{\langle p|\psi\rangle = \frac{1}{(\pi K)^{1/4}}\, e^{-p^2/2K}}$$

(d) what is the expectation value of p^2? -- I assume they mean in x-space

$$\langle p^2 \rangle = -|N|^2 \int_{-\infty}^{\infty} e^{-Kx^2/2} \frac{\partial^2}{\partial x^2} e^{-Kx^2/2}\, dx = -|N|^2 \int_{-\infty}^{\infty} e^{-Kx^2/2} \frac{d}{dx}(-Kx)\, e^{-Kx^2/2}\, dx$$

$$= |N|^2 K \int_{-\infty}^{\infty} e^{-Kx^2/2} \left(e^{-Kx^2/2} - Kx^2 e^{-Kx^2/2} \right) dx$$

$$= |N|^2 K \int_{-\infty}^{\infty} e^{-Kx^2} (1 - Kx^2)\, dx = |N|^2 K \left[\sqrt{\frac{\pi}{K}} - K \frac{\sqrt{\pi}}{2K^{3/2}} \right]$$

$$= |N|^2 K \left(\frac{1}{2}\sqrt{\frac{\pi}{K}} \right) = \frac{1}{2}\sqrt{\frac{K}{\pi}}\, K \sqrt{\frac{\pi}{K}} = \frac{K}{2}$$

$$\therefore \quad \boxed{\langle p^2 \rangle = \frac{K}{2}}$$

(e) The hamiltonian is

$$H = \frac{p^2}{2m} + V(x)$$

what is the potential $V(x)$?

the wave function given is the ground state wave function for a harmonic oscillator with $K \to m\omega$ in this case. So,

$$\boxed{V(x) = \frac{1}{2} K x^2 = \frac{m\omega}{2} x^2}$$