Spring 2004. #4

$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$ $\quad\quad \hbar = 1$

$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$

$H|\nu_1\rangle = \sqrt{p^2c^2 + m_1^2c^4}|\nu_1\rangle$

$H|\nu_2\rangle = \sqrt{p^2c^2 + m_2^2c^4}|\nu_2\rangle$

$|\psi(0)\rangle = |\nu_\mu\rangle$

$|\psi(t)\rangle = -e^{-i\sqrt{p^2c^2+m_1^2c^4}\,t}\sin\theta|\nu_1\rangle + e^{-i\sqrt{p^2c^2+m_2^2c^4}\,t}\cos\theta|\nu_2\rangle$

in limit where $ct = L$

$\langle\nu_e|\psi(t)\rangle = -\sin\theta\cos\theta\, e^{-i\sqrt{p^2c^2+m_1^2c^4}\,t} + \sin\theta\cos\theta\, e^{-i\sqrt{p^2c^2+m_2^2c^4}\,t}$

$= \sin\theta\cos\theta\left(-e^{-iL\sqrt{p^2+m_1^2c^2}} + e^{-iL\sqrt{p^2+m_2^2c^2}}\right)$

But $\quad iL\sqrt{p^2+m^2c^2} = iLp\sqrt{1+\frac{m^2c^2}{p^2}} \approx 1 + \frac{m^2c^2}{2p^2}$

$= \sin\theta\cos\theta\left(e^{-iLp}\left(-e^{-iLp\frac{m_1^2c^2}{2p^2}} + e^{-iLp\frac{m_2^2c^2}{2p^2}}\right)\right)$

$P(t \to L) = |\langle\nu_e|\psi(t)\rangle|^2 = \sin^2\cos^2\theta\left(-e^{iLp\frac{m_1^2c^2}{2p^2}} + e^{iLp\frac{m_2^2c^2}{2p^2}}\right)\left(-e^{-iLp\frac{m_1^2c^2}{2p^2}} + e^{-iLp\frac{m_2^2c^2}{2p^2}}\right)$

$= \sin^2\theta\cos^2\theta\left(+1 + 1 - e^{-\frac{iLpc^2}{2p^2}(m_2^2-m_1^2)} - e^{-\frac{iLpc^2}{2p^2}(m_1^2-m_2^2)}\right)$

$= \sin^2\theta\cos^2\theta\left(-e^{+\frac{iLc^2}{2p}\Delta m^2} - e^{-\frac{iLc^2}{2p}\Delta m^2} + 2\right)$

$\Delta m^2 = m_1^2 - m_2^2$

$= \sin^2\theta\cos^2\theta\left(2 - 2\cos\left(\frac{Lc^2\Delta m^2}{2p}\right)\right]$

$= \frac{\sin^2(2\theta)}{4}\,4\,\sin\left(\frac{Lc^2\Delta m^2}{4p}\right)$

$\Downarrow$

$$P(t) = \sin^2(2\theta)\,\sin^2\left(\frac{L\,c^2\Delta m^2}{4p}\right)$$