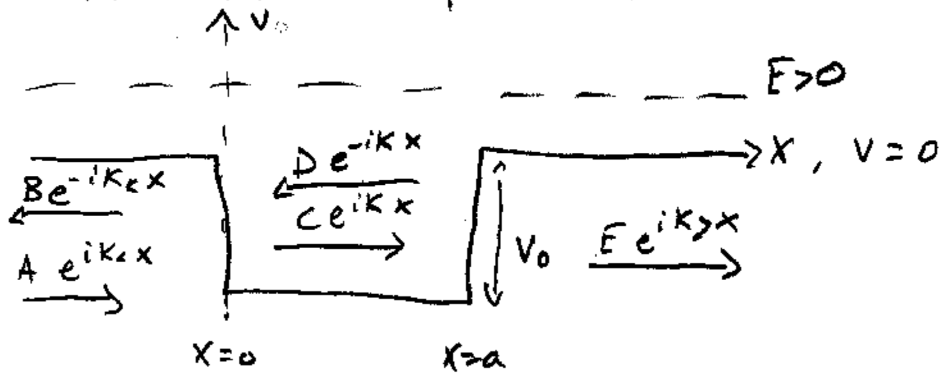
Calculate the transmission coefficient for a particle of energy $E>0$ scattering off the 1D potential well

$$V(x) = \begin{cases} V_0 & 0 < x < a \\ 0 & \text{elsewhere} \end{cases}$$

where $V_0 < 0$. Are there resonance phenomena?



Solutions are shown on the figure above where

$$K_<^2 = K_>^2 = 2mE$$

$$K^2 = 2m(E + |V_0|)$$

Boundary conditions ($\psi(x)$ and $\frac{d\psi}{dx}$ are continuous at $x=0$ and $x=a$) yields

at $x=0$: $\quad A + B = C + D \quad$ and $\quad K_<(A - B) = K(C - D)$

at $x=a$: $\quad Ce^{iKa} + De^{-iKa} = Ee^{iK_<a} \quad$ and $\quad K(Ce^{iKa} - De^{-iKa}) = K_< E e^{iK_<a}$

solving this system of equations for E yields (see Zettili p 214-215)

$$E = 4K_<K\,A e^{-iK_<a}\left[4K_<K\cos(Ka) - 2i(K_<^2 + K^2)\sin(Ka)\right]^{-1}$$

Since the transmission coefficient is defined as

$$T = \frac{K_<|E|^2}{K_<|A|^2} = \left[1 + \frac{1}{4}\left(\frac{K_<^2 - K^2}{K_<K}\right)^2 \sin^2(Ka)\right]^{-1}$$

Now let's substitute in for $K_<$ and K.

$$T = \left[1 + \frac{1}{4} \left(\frac{2mE - 2mE - 2m|V_0|}{\sqrt{2mE(2mE + 2m|V_0|)}} \right)^2 \sin^2\left[\sqrt{2m(E+|V_0|)}\,a\right] \right]^{-1}$$

$$= \left[1 + \frac{1}{4} \left(\frac{4m^2 V_0^2}{2mE(2mE + 2m|V_0|)} \right) \sin^2\left[\sqrt{2m(E+|V_0|)}\,a\right] \right]^{-1}$$

$$\boxed{T = \left[1 + \frac{|V_0| \sin^2\left[\sqrt{2m(E+|V_0|)}\,a\right]}{4E\left[1 + \left(\frac{E}{|V_0|}\right)\right]} \right]^{-1}}$$

resonance phenomenon occur when the maxima of the transmission coefficient coincides with the energy eigenvalues. This does not occur classically ... it results from a constructive interference between the incident and reflected waves. This phenomenon is observed experimentally when low-energy ($E \sim 0.1\,eV$) electrons scatter off noble atoms (Ramsauer-Townsend effect) and neutrons off nuclei.

So,

when $T = 1$, we have resonance. This occurs when $\sin^2[\,] = 0$

Thus, when

$$\boxed{a\sqrt{2m(E+|V_0|)} = n\pi} \quad , n = 0, 1, 2, 3, \ldots$$