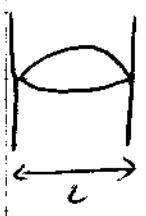


Stat. Mech. S'04 #6; S'01 #13

For relativistic bosons

$$E = |\vec{p}|c$$

a) First we need the density of states $D(E)$ for 3-D:



$$\frac{L}{\lambda} = n \Rightarrow \frac{\partial L}{\partial n} = \lambda; \quad p = \frac{h}{\lambda} = \frac{h}{\partial L} n$$

$$E = c|\vec{p}| = c(\vec{p}_x^2 + \vec{p}_y^2 + \vec{p}_z^2)^{1/2} = \frac{hc}{2L} (\pi^2 x^2 + \pi^2 y^2 + \pi^2 z^2)^{1/2}$$

$$= \frac{hc}{2L} n \Rightarrow n = \frac{2L}{hc} E \Rightarrow dn = \frac{2L}{hc} dE$$

$$\frac{(\lambda s + 1)}{8} \int_0^\infty 4\pi n^2 dn = \frac{(\lambda s + 1)}{8} \int_0^\infty 4\pi \left(\frac{2L}{hc}\right)^3 E^2 dE = \int_0^\infty \underbrace{\frac{(\lambda s + 1) 4\pi V}{(hc)^3}}_{D(E)} E^2 dE$$

$$D(E) = \frac{(\lambda s + 1) 4\pi V}{(hc)^3} E^2$$

The condition for BEC is determined by the boson temperature T_B , which can be derived followingly:

$$\int_0^\infty \frac{1}{e^{E/KT} - 1} D(E) dE = N \text{ for } T = T_B$$

$$\Rightarrow \frac{(\lambda s + 1) 4\pi V}{(hc)^3} \int_0^\infty \frac{E^2}{e^{E/KT} - 1} dE = \frac{(\lambda s + 1) 4\pi V (KT)^3}{(hc)^3} \int_0^\infty \frac{x^2}{e^x - 1} dx$$

$$x = E/KT \Rightarrow E = KTx \Rightarrow dE = KTx dx \quad \quad \quad = 2.404$$

hence

$$\frac{(2s+1)4\pi V (kT_B)^3}{(hc)^3} 2.404 = N$$

$$\Rightarrow (kT_B)^3 = \frac{N}{V} \frac{(hc)^3}{(2s+1)4\pi \cdot 2.404}$$

$$\Rightarrow T_B = \left(\frac{N}{k^3 V} \frac{(hc)^3}{(2s+1)4\pi \cdot 2.404} \right)^{1/3}$$

b) yes it does occur - just derive $D(E)$ for 2-D case and repeat above steps:

$$E = \frac{hc}{\lambda c} (\pi x^2 + \pi y^2) = \frac{hc}{\lambda c} n \Rightarrow n = \frac{\lambda L}{hc} E \Rightarrow dn = \frac{\lambda L}{hc} dE$$

$$\frac{(2s+1)}{4} \int_0^\infty 2\pi \lambda dn = \frac{(2s+1)}{4} \int_0^\infty 2\pi \left(\frac{\lambda L}{hc} \right) E dE = \int_0^\infty \underbrace{\frac{(2s+1)2\pi A}{(hc)^2}}_{D(E)} E dE$$

$$D(E) = \frac{(2s+1)2\pi A}{(hc)^2} E$$

$$\int_0^\infty \frac{(2s+1)2\pi A}{(hc)^2} \frac{E}{e^{E/kT}} dE = \frac{(2s+1)2\pi A}{(hc)^2} (kT)^2 \int_0^\infty \frac{x}{e^x - 1} dx = N$$

$$x \equiv E/kT \Rightarrow E = kTx \Rightarrow dE = kT dx$$

$$\frac{\pi^2}{6}$$

so

$$(kT_B)^2 = \frac{N}{A} \frac{3(2s+1)}{\pi^3 (hc)^2} \Rightarrow T_B = \left(\frac{N}{k^2 A} \frac{3(2s+1)}{\pi^3 (hc)^2} \right)^{1/2}$$

c) BEC does not occur in 1-D case.

$$E = c|\vec{p}| = \frac{\hbar c}{\lambda} n \Rightarrow n = \frac{\lambda L}{\hbar c} E \Rightarrow dn = \frac{\lambda L}{\hbar c} dE$$

$$\frac{(\lambda S+1)}{2} \int_0^{\infty} dn = \int_0^{\infty} \underbrace{\frac{(\lambda S+1) \lambda L}{\hbar c}}_{D(E)} dE$$

$$D(E) = \frac{(\lambda S+1) L}{\hbar c}$$

$$\int_0^{\infty} \frac{(\lambda S+1) L}{\hbar c} \frac{dE}{e^{-E/KT} - 1} = \frac{(\lambda S+1) L}{\hbar c} kT \int_0^{\infty} \frac{dx}{e^x - 1} = \text{undefined.}$$

$$x \equiv E/KT \Rightarrow dE = kT dx \quad \text{undefined}$$