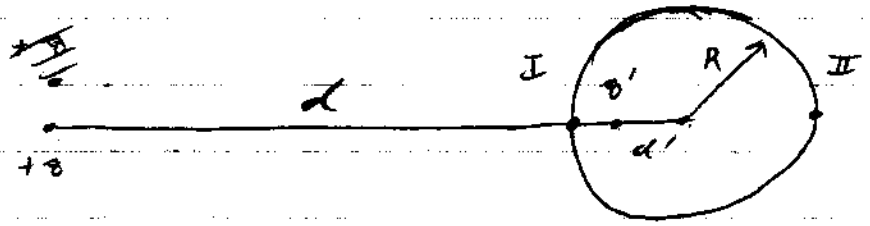
EM S'04 # 8



Step 1 is to determine q' & d':

At I & II $V=0$ for grounded conductor:

	q	q'
I	$d-R$	$R-d'$
II	$d+R$	$R+d'$

I $\Rightarrow$ $\frac{q}{d-R} + \frac{q'}{R-d'} = 0$ (1)

II $\Rightarrow$ $\frac{q}{d+R} + \frac{q'}{R+d'} = 0$ (2)

(1): $\frac{q}{d-R} = \frac{-q'}{R-d'} \Rightarrow q = -q'\frac{(d-R)}{R-d'}$

plug into (2):

$$\frac{-q'(d-R)}{(R-d')(d+R)} = \frac{-q'}{R+d'} \Rightarrow (d-R)(R+d') = (R-d')(d+R)$$

$$dR + dd' - R^2 - Rd' = dR + R^2 - dd' - d'R$$

$$\Rightarrow 2dd' = 2R^2 \Rightarrow \boxed{d' = \frac{R^2}{d}}$$

plug this back into (1)

$$q' = -\frac{q(R-d')}{(d-R)} = -\frac{q\left(R-\frac{R^2}{d}\right)}{(d-R)} \times \frac{d}{d} = -q\frac{(Rd-R^2)}{d(d-R)} = \frac{-qR}{d}\frac{(d-R)}{(d-R)}$$

$$\boxed{q' = -\frac{qR}{d}}$$

Now for the force between q & q'

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq'}{(d-d')^2} = \frac{-q^2}{4\pi\epsilon_0}\frac{R}{d\left(d-\frac{R^2}{d}\right)^2} = \boxed{\frac{-q^2}{4\pi\epsilon_0}\frac{Rd}{(d^2-R^2)^2}}$$

Now we place a charge $Q - q'$ on the conductor (after removing the ground). Then the force on $+q$ will be the old force plus the new force due to the charged sphere:

$$F = F_{old} + F_{new} = -\frac{q^2 R a}{4\pi\epsilon_0 (a^2 - R^2)^2} + \frac{q}{4\pi\epsilon_0}\frac{(Q - q')}{a^2}$$

$$= -\frac{q^2 R a}{4\pi\epsilon_0 (a^2 - R^2)^2} + \frac{q}{4\pi\epsilon_0}\frac{(Q + \frac{qR}{a})}{a^2}$$

Now we want the force on $+q$ to be 0:

$$\frac{q^2 R a}{4\pi\epsilon_0 (a^2 - R^2)^2} = \frac{q}{4\pi\epsilon_0}\frac{(Q + \frac{qR}{a})}{a^2} \Rightarrow \frac{qRa}{(a^2 - R^2)^2} = \frac{Q}{a^2} + \frac{qR}{a^3}$$

$$\Rightarrow \quad Q = a^2\left[\frac{qRa}{(a^2 - R^2)^2} - \frac{qR}{a^3}\right] = a^4\left[\frac{qRa^4 - qR(a^2 - R^2)^2}{a^5 (a^2 - R^2)^2}\right]$$

$$= \frac{qRa^4 - qR(a^4 - 2a^2R^2 + R^4)}{a(a^2 - R^2)^2} = \frac{qRa^4 - qRa^4 + 2qa^2R^3 - qR^5}{a(a^2 - R^2)^2}$$

$$\boxed{Q = q\left[\frac{2a^2R^3 - R^5}{a(a^2 - R^2)^2}\right]}$$