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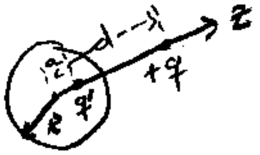
A point charge q is located a distance d from the center of a conducting sphere of radius R. What must the total charge on the conducting sphere be for the force on the point charge to be zero?



we know that with conducting image charge problems with spheres that the location, a, of the image charge and charge, q', is

$$a = \frac{R^2}{d} \quad \text{and} \quad q' = -q\frac{R}{d}$$

→ this applies when the sphere is grounded to have $V=0$ on surface of sphere.

Now the force corresponding to the sphere if grounded is (see Fall 2002 #10(c))

$$F = \frac{qq'}{|d-a|^2} = \frac{-q^2(R/d)}{|d-\frac{R^2}{d}|^2} = \frac{-q^2Rd}{|d^2-R^2|^2}$$

Now, if sphere is not grounded, there is some charge on the surface, $Q-q'$. (see Griffiths' problem 3.8 for a similar problem). So, now the total charge on the surface of the sphere is $(Q-q')+q' = Q$. Then the force on the charge q is

$$F = \frac{-q^2Rd}{|d^2-R^2|^2} + \frac{q(Q-q')}{d^2} \quad , \; q' = -q\frac{R}{d}$$

Setting this force equal to zero and solving for Q yields

$$\frac{q^2Rd}{|d^2-R^2|^2} = \frac{qQ}{d^2} + \frac{q^2R}{d^3}$$

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$$\Rightarrow \quad Q = \frac{d^2}{q}\left[\frac{q^2 R d}{|d^2-R^2|^2} - \frac{q^2 R}{d^3}\right]$$

$$= q\left[\frac{R d^3 \cdot d^3}{d^3 |d^2-R^2|^2} - \frac{d^2 R (d^2-R^2)^2}{d^3 |d^2-R^2|^2}\right]$$

$$= qR\left[\frac{d^6 - d^6 - R^4 d^2 + 2d^4 R^2}{d^3 |d^2-R^2|^2}\right]$$

Thus, the charge must be

$$\boxed{Q = qR\left[\frac{2d^2R^2 - R^4}{d(d^2-R^2)^2}\right]}$$