Spring 2005 #1

$H' = -qEx$

a) Calculate correction to second order pertubation theory

$H = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{m\omega^2 x^2}{2}\psi$ or $\frac{-1}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{m\omega^2x^2}{2}\psi$ in natural units.

$E = \hbar\omega(n+\tfrac{1}{2})$

$|\psi\rangle = |n\rangle$

1st $\langle n|H'|n\rangle = -qE\langle n|x|n\rangle$ $\qquad x = \sqrt{\frac{\hbar}{2m\omega}}(a+a^\dagger)$

$= -\frac{eE}{\sqrt{2m\omega}}[\underbrace{\langle n|a|n\rangle}_{\cancel{\langle n|n-1\rangle}\,0} + \underbrace{\langle n|a^\dagger|n\rangle}_{\cancel{\langle n|n+1\rangle}\,0}]$ $\qquad a|n\rangle = \sqrt{n}|n-1\rangle$, $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$

$= 0$

2nd $\sum_{m\neq n}\frac{|\langle m|H'|n\rangle|^2}{E_n^0 - E_m^0}$

$\langle m|H'|n\rangle = 0$ onles $m = n+1$ or $m-1$

$\langle n+1|H'|n\rangle = -\frac{eE}{\sqrt{2m\omega}}[\cancel{\langle n+1|a|n\rangle} + \langle n+1|a^\dagger|n\rangle]$

$= \frac{-eE}{\sqrt{2m\omega}}\sqrt{n}$

$\langle n-1|H'|n\rangle = \frac{-eE}{\sqrt{2m\omega}}\sqrt{n+1}$

$\frac{\frac{e^2E^2 n}{2m\omega}}{E_n^0 - E_{n+1}} + \frac{\frac{e^2E^2(n+1)}{2m\omega}}{E_n^0 - E_{n+1}} = \frac{\frac{e^2E^2n}{2\omega\omega}}{\omega} + \frac{\frac{e^2E^2(n+1)}{2m\omega}}{\omega}$

$\omega(n+\tfrac{1}{2}) - \omega(n+1+\tfrac{1}{2})$

$\omega n + \frac{\omega}{2} - \omega n - \omega/2 - \omega$

$$E^{(2)} = -\frac{q^2 E^2}{2m\omega^2}$$

b) Find Exact energy

$$-\frac{1}{2m}\frac{d^2\psi}{dx^2} + \underbrace{\frac{m\omega^2 x^2}{2}\psi - qEx\psi}_{} = E\psi$$

rewrite to make it look like the harmonic oscillator

$$\frac{m\omega^2}{2}\left[x^2 - \frac{2qE}{m\omega^2}x\right] = \frac{m\omega^2}{2}\left[x^2 - \frac{2qEx}{m\omega^2} + \left(\frac{qE}{m\omega^2}\right)^2 - \left(\frac{qE}{m\omega^2}\right)^2\right]$$

$$= \frac{m\omega^2}{2}\left[x - \frac{qE}{m\omega^2}\right]^2 - \underbrace{\frac{m\omega^2}{2}\frac{q^2E^2}{m^2\omega^4}}_{\frac{q^2E^2}{2m\omega^2}}$$

$$-\frac{1}{2m}\frac{d^2\psi}{dx^2} + \frac{m\omega^2}{2}\left(x - \frac{qE}{m\omega^2}\right)^2\psi = \underbrace{\left(E + \frac{q^2E^2}{2m\omega^2}\right)}_{E'}\psi$$

$U = x - \frac{qE}{m\omega^2}$ $\quad dU = dx$

$$-\frac{1}{2m}\frac{d^2\psi}{dU^2} + \frac{m\omega^2 U^2}{2}\psi = E'\psi \Rightarrow E' = \omega(n+1/2) = E + \frac{q^2E^2}{2m\omega^2}$$

$$E = \omega(n+1/2) - \frac{q^2E^2}{2m\omega^2} = E^0 + E^2$$